Faculty EEMCS		System theory (TW2530)
DIAM	ONLINE EXAM	13.30 – 16.30 hours
TU Delft		03-07-2020

To participate in this exam, you must subscribe the following declaration.

I declare that I have made this examination on my own, with no assistance and in accordance with the TU Delft policies on plagiarism, cheating and fraud.

If you agree, **write the above declaration on top of your work**, and validate it with your **signature**. If you don't agree, your work will not be assessed.

- Work tidy and write down your answers clearly. Do have too many scratchings!
- On each page of your work, there should be a page number and your student number.
- When the time for the examination has passed, i.e., at 16.30 hours, stop with the exam, make photos/scans of your work, try to combine them into one pdf, and upload the document through the Assigments module of Brightspace.
- You have **15 minutes** to make photos/scans of your work and upload the document in Brightspace.
- If making one pdf is too complicated, you can submit the separate pages as well.
- In case of urgent issues, you can contact me at: `j.w.vanderwoude@tudelft.nl`, or 06-48934673.

This exam consists of six questions. For each question, explain precisely how you have obtained your answer. Answers that are not properly explained will be not rewarded.

Some of the questions contain a parameter depending on your seven digit student number. Carefully determine the requested parameter value from your student number! Write the obtained parameter value on top of your answer to the corresponding question!

1. Let Γ be the sum of the last four digits of your student number. Use the value of Γ in this exercise and write its value on top of your answer.

Consider the nonlinear system $\dot{x} = f(x,u),\ y = g(x,u)$ with $x \in \mathbb{R}^3$, $u, y \in \mathbb{R}$, where

$$f(x,u) = \begin{pmatrix} e^{x_2}x_1 - \Gamma x_3 \\ -\sin(x_2) \\ \Gamma x_3 - x_1(u + x_2) \end{pmatrix}, \quad g(x,u) = (x_1 - 3)x_3 - x_2 u, \quad \text{when } x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}.$$

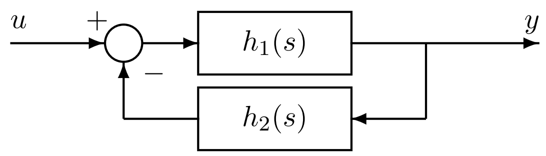
(a) Show that the pair $(\tilde{x}, \tilde{u})$ with $\tilde{x}(t) = \begin{pmatrix} \Gamma \\ 0 \\ 1 \end{pmatrix}$, $\tilde{u}(t) = 1$, for all $t \geq 0$, satisfies $\dot{x} = f(x,u)$.

(b) Next to the solution pair $(\tilde{x}, \tilde{u})$ given in part (a), there are also other solution pairs. Give an example of another solution pair with a nonzero state and a nonzero control.

(c) Compute the linearisation of the above nonlinear system round the solution pair $(\tilde{x}, \tilde{u})$ given in part (a).

2. Let Λ be the sum of the first three digits of your student number. Use the value of Λ in this exercise and write its value on top of your answer.

Consider the feedback system



with $h_1(s) = \Lambda(\frac{1}{s+1} - \frac{1}{s}) + \frac{s+k}{s(s+1)}$, and $h_2(s) = \frac{4}{s+2}$, where k is a real constant.

(a) Give state space descriptions of $h_1(s)$.

(b) Find the transfer function of the feedback system. Also give a state space description.

(c) For which values of k is the system stable?

3. Let Ω be the sum of the middle three digits of your student number. Use the value of Ω in this exercise and write its value on top of your answer.

Consider the lineair system $\dot{x} = Ax + Bu,\ y = Cx$ with

$$A = \begin{pmatrix} 0 & 0 & \alpha \\ 0 & \beta & 0 \\ -\alpha & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} \Omega \\ 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & \Omega & 0 \end{pmatrix},$$

where α and β are real parameters.

(a) For which values of α and β is the system is stable, asymptotically stable or unstable? In your answer be as complete as possible.

(b) Investigate the controllability of the system. Pay attention to the role of α and β.

(c) Compute the impulse response of the system.

4. Let Ψ be the sum of the middle five digits of your student number. Use the value of Ψ in this exercise and write its value on top of your answer.

Consider the linear system $\dot{x} = Ax + Bu, y = Cx$ with

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & \Psi & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \Psi & 0 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 1 & 0 & 0 \end{pmatrix}.$$

Let $q(s) = s^4 + 2s^3 - 2s - 1$ be a given polynomial.

(a) Is the system detectable? Hint, have a good look at the matrices.

(b) Find a matrix K such that the characteristic polynomial of $A - KC$ equals $q(s)$. Is such an K uniquely determined?

(c) Describe the set of all characteristic polynomials of $A - KC$ that can be obtained by choosing matrix K suitably. Give your answer with as few as possible parameters.

5. Consider the linear system $\dot{x} = Ax + Bu, y = Cx$, with

$$A = \begin{pmatrix} A_{11} & 0 \\ A_{21} & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} C_1 & C_2 \end{pmatrix},$$

where $A_{11}, A_{21}, A_{22}, B_1, C_1$ and C_2 are real matrices of dimensions $n_1 \times n_1, n_2 \times n_1, n_2 \times n_2, n_1 \times m, p \times n_1$ and $p \times n_2$, respectively. Assume that $n_1 + n_2 = n$, with $n_1 > 0$ and $n_2 > 0$, and that $m > 0, p > 0$.

(a) Prove that if the pair (A, B) is controllable, this is also true for the pairs (A_{11}, B_1) and (A_{22}, A_{21}).

(b) Show that the converse of part (a) is not necessarily true, i.e., there exist controllable pairs (A_{11}, B_1) and (A_{22}, A_{21}), such that the pair (A, B) is not controllable.

(c) Prove that the converse of part (a) is true, when additionally rank $B_1 = n_1$.

6. Let Δ be <u>two times</u> the sum of the first four digits of your student number. Use the value of Δ in this exercise and write its value on top of your answer.

Consider the following statements.

(a) If the discrete-time system $x(k+1) = Ax(k)$ is asymptotically stable, then so is the discrete-time system $x(k+1) = A^{\Delta}x(k)$.

(b) Let A be an $n \times n$ matrix and C be an $p \times n$ matrix. If rank A + rank $C > n$, then (C, A) always is an observable pair.

(c) The controllable subspace corresponding to the pair (A^{Δ}, B) is A-invariant.

For each of the above statements, investigate whether the statement is true or false. Motivate your conclusion by means of a simple reasoning, calculation or (counter)example.

question	1	2	3	4	5	6
points	4	6	8	6	6	6

Each subquestion is worth 2 points.

The grade for the exam is the sum of the obtained points, divided by four, plus one.

The final grade for this course is the total of

80% of the grade for the exam 20% of the grade for the case combined with the bonus for the homework as written on Brightspace