Faculty EEMCS	System theory (TW2530)
DIAM	9.00 – 12.00 hours
TU Delft	31-05-2018

This exam consists of five questions. For each question, explain precisely how you have obtained your answer. Answers that are not properly explained will be not rewarded.

1. Consider a combination of two coupled vessels and a pump. The relevant equations are given as follows

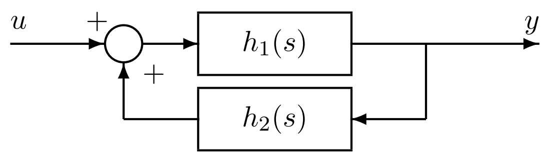
$$\begin{cases} \dot{x}_1 &= \alpha\sqrt{x_2} - \alpha\sqrt{x_1} \\ \dot{x}_2 &= \beta x_3 - \alpha\sqrt{x_2} \\ \dot{x}_3 &= -\gamma x_3 + \delta u \end{cases} \qquad \text{and} \qquad y = x_2.$$

The variables x_1 and x_2 denote the (positive) fluid levels in the two vessels, and x_3 is the 'flow rate' by the pump. The variable u stands for the electrical current supplied to the pump motor and y represents the output variable. All variables have been made dimensionless. Further, α, β, γ and δ denote parameters with a positive value. In this question, take $\alpha = 2, \beta = 5, \gamma = 2$ and $\delta = 1$.

(a) Think of the above equations as a system of the form $\dot{x} = f(x, u),\ y = g(x, u)$, and compute the stationary (= constant) state $x(t) \equiv \tilde{x}$ and input $u(t) \equiv \tilde{u}$, when x_1 has a constant value equal to 25 (meaning that $x_1(t) = 25$ for all t).

(b) Determine the linearization of the above system around the solution pair $(\tilde{x}, \tilde{u})$ computed in part (a). Don't forget the output equation.

2. Consider the following feedback interconnection of two systems



with $$h_1(s) = \frac{s-1}{s(s+1)} \quad \text{and} \quad h_2(s) = \frac{k}{s+2},$$

where k is a real constant. (Note that the input to first system is the <u>sum</u> of u and the output of second system.)

(a) What is the transfer function of the feedback interconnection? Also give a description in state space form.

(b) For which values of k is the feedback interconnection stable?

3. Consider the linear system

$$\dot{x} = Ax + Bu, \quad y = Cx$$

with

$$A = \begin{pmatrix} 1 & 1 & -1 & \alpha \\ \beta & 0 & -\beta & 0 \\ 1 & 0 & -1 & \alpha \\ 0 & 2 & 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & -1 & 0 \end{pmatrix},$$

where α en β are real parameters.

(a) Is the system controllable? What is the controllable subspace? Does this subspace depend on α and β?

(b) Show that the system is not observable. Compute the non-observable subspace. Does this subspace depend on α and β?

(c) Determine a decomposition of the system in a controllable part and a part that is not controllable. Don't forget the output equation.

(d) Compute the impulse response of the system using the results obtained in part (c). Can you explain your result in the light of the subspaces computed in parts (a) and (b)?

(e) Investigate the (state space) stability of the system for $\alpha = 0$ and $\beta = 0$.

4. Consider the linear system

$$\dot{x} = Ax + Bu, \quad y = Cx$$

with

$$A = \begin{pmatrix} -2 & 0 & 0 \\ -3 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}.$$

(a) Using the appropriate Hautus tests, show that the system is stabilizable and detectable.

(b) Compute a matrix F such that all eigenvalues of $A + BF$ have a negative real part. Be specific on where the eigenvalues of $A + BF$ are located!

(c) Compute a matrix K such that all eigenvalues of $A - KC$ are located at $-1, -2, -3$. Notice that with some insight hardly any calculation is required!

(d) Take the results of parts (b) and (c), and combine them to obtain a dynamic controller such that the combination of system and controller has eigenvalues in the open left half of the complex plane. Where precisely are these eigenvalues located?

5. Consider the following statements:

(a) *Let A be an $n \times n$ matrix and B an $n \times m$ matrix. If the pair (A, B) is stabilizable, then also the pair (A^2, B) is stabilizable.*

(b) *There exists a system $\dot{x} = Ax + Bu$, $y = Cx$, with A an $n \times n$ matrix, B an $n \times 1$ matrix, C a $1 \times n$ matrix and an appropriate initial state $x(0) = x_0 \in \mathbb{R}^n$, such that $y(t) = \sin 2t$, while $u(t) = e^{-t}$.*

(c) *The series connection of a stable system and an unstable system is always unstable.*

(d) *The characteristic polynomial of the matrix A defined by*

$$A = \begin{pmatrix} 0 & \cdots & \cdots & 0 & -p_0 \\ 1 & \ddots & & \vdots & -p_1 \\ 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & -p_{n-2} \\ 0 & \cdots & 0 & 1 & -p_{n-1} \end{pmatrix}$$

is equal to $s^n + p_{n-1}s^{n-1} + \cdots + p_1 s + p_0$.

For each of the above statements, investigate whether the statement is true or false. Motivate your conclusion by means of a simple reasoning, calculation or (counter)example.

Points per question

question	1	2	3	4	5
points	1.0	1.5	2.5	2.0	2.0

The grade for the exam is the sum of the obtained points plus one.

The final grade for this course is the total of

80% of the grade for the exam
20% of the grade for the case
combined with the bonus for the
homework as written on Brightspace