Faculty EEMCS	System theory (TW2530)
DIAM	13.30 – 16.30 hours
TU Delft	08-04-2019

This exam consists of six questions. For each question, explain precisely how you have obtained your answer. Answers that are not properly explained will be not rewarded.

1. Consider a process with a scalar input u and a scalar output y. Let the relationship between u, y and an auxiliary variable v be given by means of the following equations.

$$\ddot{v}(t) - \dot{v}^3(t) + v(t) = u^3(t), \quad y(t) = v^2(t) + u^2(t).$$

(a) Write the process as a system of the form $\dot{x} = f(x, u),\ y = g(x, u)$, where you have to choose your own state x.

(b) Find $u(t)$ and $y(t)$ when it it given that $v(t) = 2 \sin t$ is a solution of the differential equation.

(c) Derive the linearisation of the system around the solution obtained in part (b).

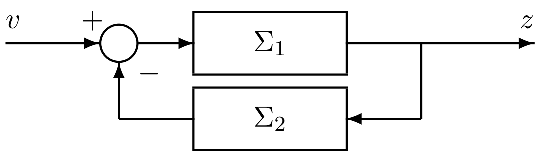
2. Given are the system Σ_1 by means of its transfer function

$$\frac{s+4}{s^2+3s+4},$$

and the system Σ_2 by means of the state space description

$$\dot{x} = \begin{pmatrix} 0 & 1 \\ -4 & -5 \end{pmatrix} x + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u, \quad y = \begin{pmatrix} \alpha & 3 \end{pmatrix} x,$$

where α is a real parameter. The two systems are interconnected as follows.



(a) Find the transfer function from v to z of the interconnected system. Keep your calculations transparent!

(b) For which values of α is the interconnected system stable?

(c) For $\alpha = 3$, find a state space description of the interconnected system with a minimal state space dimension. Explain how you check whether this dimension is minimal.

3. Consider the lineair system $\dot{x} = Ax + Bu$, $y = Cx$ with

$$A = \begin{pmatrix} 0 & \beta^2 & 0 \\ 0 & 0 & -\frac{1}{2} \\ 2 & 2\beta^2 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \quad C = \begin{pmatrix} 2 & 1 & 4 \end{pmatrix},$$

where β is a real parameter.

(a) Investigate if the system is stable, asymptotically stable or unstable, and whether this depends on β.

(b) Show that the system is not controllable, independently of the value of β.

(c) Compute a decomposition of the system in a controllable part and a part that is not controllable. Clearly indicate the individual parts.

4. Consider the linear system $\dot{x} = Ax + Bu$ with

$$A = \begin{pmatrix} 0 & 1 & 5 & 0 \\ -1 & 3 & 0 & 0 \\ 0 & 0 & -1 & -4 \\ 0 & 0 & 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

Let $q(s) = s^4 + 6s^3 + 16s^2 + 26s + 15$ be a given polynomial.

(a) Is the system stabilizable? Hint, have a good look at the matrices.

(b) Find a matrix F such that the characteristic polynomial of $A + BF$ equals $q(s)$.

(c) Describe the set of all characteristic polynomials of $A + BF$ that can be obtained by choosing matrix F suitably.

5. Consider the linear system $\dot{x} = Ax + Bu, y = Cx$, with

$$A = \begin{pmatrix} A_{11} & 0 \\ 0 & A_{22} \end{pmatrix}, \quad B = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}, \quad C = \begin{pmatrix} C_1 & C_2 \end{pmatrix},$$

where $A_{11}, A_{22}, B_1, B_2, C_1$ and C_2 are real matrices of dimensions $n_1 \times n_1, n_2 \times n_2, n_1 \times m, n_2 \times m, p \times n_1$ and $p \times n_2$, respectively. Assume that $n_1 + n_2 = n$, with $n_1 > 0$ and $n_2 > 0$, and that $m > 0, p > 0$.

(a) Prove that if the pair (C, A) is observable, this is also true for the pairs (C_1, A_{11}) and (C_2, A_{22}).

(b) Show that the converse of part (a) is not necessarily true, i.e., there exist observable pairs (C_1, A_{11}) and (C_2, A_{22}), such that the pair (C, A) is not observable.

(c) Prove that the converse of part (a) is true, when additionally the matrices A_1 and A_2 have disjoint sets of eigenvalues.

6. Consider the following statements.

(a) Consider the linear system $\dot{x} = Ax + Bu, y = Cx$, where A, B and C are real matrices of dimensions $n \times n, n \times m$ and $p \times n$, respectively. The impulse response of the system can be computed using only the products CA^kB, for $k = 0, 1, \ldots, n-1$.

(b) Let A be an $n \times n$ matrix and B be an $n \times m$ matrix. If rank A + rank $B < n$, then (A, B) can never be a controllable pair.

(c) There does not exist a $p \times n$ matrix C and an invertible $n \times n$ matrix A such that the pairs (C, A) and (C, A^{-1}) are both discrete time detectable.

For each of the above statements, investigate whether the statement is true or false. Motivate your conclusion by means of a simple reasoning, calculation or (counter)example.

question	1	2	3	4	5	6
points	6	6	6	6	6	6

Each subquestion is worth 2 points.

The grade for the exam is the sum of the obtained points, divided by four, plus one.

The final grade for this course is the total of

80% of the grade for the exam 20% of the grade for the case combined with the bonus for the homework as written on Brightspace