Final Exam: Linear Algebra CSE1205
April 17, 2019, 13:30 – 16:30 hs

- Calculators and formula sheets are **not** allowed.
- Credits: 2 points for questions from Part I (except question 17 and 18; 1 point for these) and 5 points for questions from Part II.
- The final score: Sum and divide by 6.

PART I: MULTIPLE CHOICE QUESTIONS

1. How many solutions does the following system of equations has?

$$\begin{aligned} x_1 + \ x_2 + \ x_3 &= 6 \\ 2x_1 + \ x_2 + 3x_3 &= 10 \\ x_1 + 3x_2 + 2x_3 &= 13 \end{aligned}$$

A. No solution **B.** ∞ many solutions
C. A unique solution **D.** None of the other statements apply

2. Consider the vectors $\mathbf{b}_1 = \begin{bmatrix} 1 & 1 & -1 & 2 \end{bmatrix}^T$, $\mathbf{b}_2 = \begin{bmatrix} -1 & 3 & 0 & 1 \end{bmatrix}^T$, $\mathbf{b}_3 = \begin{bmatrix} 3 & -1 & -2 & 3 \end{bmatrix}^T$. The dimension of $\text{Span}\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ is:

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4

3. It is given that $AB = \begin{bmatrix} 1 & 5 \\ 2 & 6 \\ 3 & 7 \\ 4 & 8 \end{bmatrix}$, where $B = \begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}$. The (3,2)-entry a_{32} of A is equal to:

A. -17 **B.** -13 **C.** -7 **D.** -1 **E.** 1 **F.** 7 **G.** 13 **H.** 17

4. Suppose the equation $A\mathbf{x} = \mathbf{b}$, for an $n \times n$ matrix A, is inconsistent for *some* $\mathbf{b}$ in $\mathbb{R}^n$. Which of the following statements *must* be true?

A. $\det A \neq 0$ **B.** The columns of A are linearly independent
C. $\text{Col}\, A = \mathbb{R}^n$ **D.** $A\mathbf{x} = \mathbf{0}$ has only the trivial solution
E. A has n pivot positions **F.** The map $\mathbf{x} \mapsto A\mathbf{x}$ is one-to-one
G. $\text{Nul}\, A \neq \{\mathbf{0}\}$ **H.** None of the others

5. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that leaves $\mathbf{e}_1$ unchanged, while $\mathbf{e}_2$ is mapped to $-2\mathbf{e}_1 + \mathbf{e}_2$. Let $S : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that reflects points though the line $y = -x$. Find the standard matrix for the composition $S \circ T$:

A. $\begin{bmatrix} 2 & -1 \\ -1 & 0 \end{bmatrix}$ **B.** $\begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$ **D.** $\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$
E. $\begin{bmatrix} 1 & -1 \\ -2 & 2 \end{bmatrix}$ **F.** $\begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}$ **G.** $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$ **H.** $\begin{bmatrix} -1 & 2 \\ 0 & -1 \end{bmatrix}$

6. Which of the sets $W_1 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \ x + y + z = 2 \right\}$, $W_2 = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \ 2x - y = z,\ x = y \right\}$ is a subspace of $\mathbb{R}^3$?

A. None of them **B.** Only W_1 **C.** Only W_2 **D.** Both

7. The dimension of Nul(A), where $A = \begin{bmatrix} -2 & 2 & 3 & 1 & 0 & -2 & 0 \\ 0 & 2 & 0 & 0 & 1 & -1 & 0 \\ 1 & 0 & -1 & 3 & 2 & 0 & 1 \end{bmatrix}$, equals:

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4 **F.** 5 **G.** 6 **H.** 7

8. Let A and B be two invertible matrices, $\mathbf{v}$ a non-zero vector and λ a non-zero scalar such that $A\mathbf{v} = \lambda B\mathbf{v}$. Then:

A. λ^{-1} is an eigenvalue of $B^{-1}A$
B. λ is an eigenvalue of $A^{-1}B$
C. λ^{-1} is an eigenvalue of BA^{-1}
D. λ is an eigenvalue of AB^{-1}
E. λ^{-1} is an eigenvalue of $A^{-1}B$
F. λ is an eigenvalue of BA^{-1}
G. λ^{-1} is an eigenvalue of AB^{-1}
H. None of the other options

9. Calculate A^5, where $A = \begin{bmatrix} 0 & 1 \\ -2 & 3 \end{bmatrix}$:

A. $\begin{bmatrix} -30 & -62 \\ 31 & 63 \end{bmatrix}$ **B.** $\begin{bmatrix} 0 & 1 \\ -2 & 243 \end{bmatrix}$ **C.** $\begin{bmatrix} 63 & -31 \\ -62 & -30 \end{bmatrix}$ **D.** $\begin{bmatrix} -30 & 63 \\ -62 & 31 \end{bmatrix}$
E. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ **F.** $\begin{bmatrix} -62 & 63 \\ -30 & 31 \end{bmatrix}$ **G.** $\begin{bmatrix} 0 & 1 \\ -32 & 243 \end{bmatrix}$ **H.** $\begin{bmatrix} -30 & 31 \\ -62 & 63 \end{bmatrix}$

10. Consider the vectors $\mathbf{b}_1 = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$, $\mathbf{b}_2 = \begin{bmatrix} 4 & 0 & 0 & 0 \end{bmatrix}^T$ and $\mathbf{b}_3 = \begin{bmatrix} 0 & 0 & 1 & 1 \end{bmatrix}^T$.
If we apply the Gram-Schmidt process to $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ to obtain an orthogonal set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$, then $\mathbf{v}_3$ is (up to rescaling) equal to

A. $\begin{bmatrix} 0 & -1 & 1 & 0 \end{bmatrix}^T$
B. $\begin{bmatrix} 0 & -2 & 1 & 1 \end{bmatrix}^T$
C. $\begin{bmatrix} \frac{2}{3} & \frac{2}{3} & \frac{2}{3} & \frac{2}{3} \end{bmatrix}^T$
D. $\begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$
E. $\begin{bmatrix} 1 & 1 & -1 & -1 \end{bmatrix}^T$
F. $\begin{bmatrix} 0 & 0 & 1 & -1 \end{bmatrix}^T$
G. $\begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}^T$
H. $\begin{bmatrix} 0 & -1 & 0 & 1 \end{bmatrix}^T$

11. Determine all the distinct (real and complex) eigenvalues of the matrix
$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -2 \\ 0 & 4 & -1 \end{bmatrix}$.

A. $1, -1+2i, -1-2i$
B. 1
C. $1, 1+2i, 1-2i$
D. $1, 1+2i, -1-2i$
E. $1, -2+i, -2-i$
F. $1, 1+i, 1-i$
G. $1, 3, -1$
H. $1, 2+i, 2-i$

12. Calculate the inverse of the matrix $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, if it exists.

Then the sum of all the entries of A^{-1} is equal to:

A. -3 **B.** -2 **C.** -1 **D.** 0 **E.** 1 **F.** 2 **G.** 3 **H.** A is not invertible

13. Find the distance from $\mathbf{y} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$ to $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \right\}$:

A. 1 **B.** 2 **C.** 5 **D.** $\sqrt{29}$ **E.** $\sqrt{30}$ **F.** 10 **G.** 29 **H.** 30

For questions **14** and **15** consider the matrix $A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{bmatrix}$.

14. The algebraic multiplicity of the eigenvalue 2 of the above matrix A equals

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4 **F.** 5

15. The geometric multiplicity of the eigenvalue 2 of the above matrix A equals

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4 **F.** 5

16. Suppose the equation $A(XB^{-1})^T = C$ holds for invertible matrices A, B, C. Solving for X gives that X is equal to:

A. $C(A^{-1})^T B$ **B.** $A^{-1}CB$ **C.** $(A^{-1})^T C^T B$ **D.** $C^T A^{-1} B$
E. $(A^{-1})^T C^T B^T$ **F.** $C^T (A^{-1})^T B$ **G.** $(A^{-1})^T C B^T$ **H.** None of the above

17. Suppose that the square matrix A is row equivalent to B and $\lambda = 1$ is an eigenvalue of A. Is $\lambda = 1$ also an eigenvalue of B?

A. True **B.** False

18. Suppose that the square matrix A is row equivalent to B and $\lambda = 0$ is an eigenvalue of A. Is $\lambda = 0$ also an eigenvalue of B?

A. True **B.** False

19. Which of the following statements are always true if Q is a (not necessarily square) matrix with **orthonormal rows**?

(I) $Q^T Q = I$ (II) $QQ^T = I$

A. Both are false **B.** Only (I) is true **C.** Only (II) is true **D.** Both are true

20. Consider the following statements for a square matrix A.

(I) If A invertible and diagonalizable, then A^{-1} is also diagonalizable
(II) If A diagonalizable, then A^T is also diagonalizable

A. Both statements are false **B.** Only (I) is true
C. Only (II) is true **D.** Both statements are true

21. Find the equation $y = \beta_0 + \beta_1 x$ of the best line (in the least-squares sense) that fits the points $(0, 1), (2, 3), (4, 2)$.

A. $y = 2$ **B.** $y = 0.5 + x$ **C.** $y = 1 + x$ **D.** $y = 1.5 + 0.25x$
E. $y = 1 + 0.25x$ **F.** $y = 4 - 0.5x$ **G.** $y = 1$ **H.** $y = 1 + 0.5x$

END OF PART I.
GO TO PART II: TRUE/FALSE QUESTIONS

CSE1205 (Linear Algebra), 17 – 04 – 2019, True/False Questions

Name:	**Student ID:**
write ***readable*** *and underline your* *surname*	

You are asked to decide whether the statements are true or false.
Either give a proof or a specific counterexample.

22. If A and B are matrices such that AB exists, then: if $\mathbf{x} \in \text{Col}(AB) \implies \mathbf{x} \in \text{Col}(A)$.

23. If $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ are linearly independent vectors, then $\{\mathbf{v}_1 + \mathbf{v}_2, \mathbf{v}_2 + \mathbf{v}_3, \mathbf{v}_3 + \mathbf{v}_4, 2\mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_4\}$ are also linearly independent.

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24. If $A^T A$ is a diagonal matrix, then the columns of A are orthogonal.

25. If a square matrix A satisfies the equation $2A^2 + 3A = 4I$, then A is invertible.