Resit *Linear Algebra* (CSE1205), July 4, 2019, 13:30-16:30

- Calculators and formula sheets are **not** allowed.
- Credits: **2 points** for questions from Part I (19 questions) and **4 points** for questions from Part II (3 questions).
- The final score: Sum and divide by 5.

PART I: MULTIPLE CHOICE QUESTIONS

1. Let (x_1, x_2, x_3) be the unique solution of the system

$$\begin{aligned} x_2 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_1 \quad + \quad 5x_3 &= -2 \end{aligned}$$

Then x_1 is equal to:

A. -3 **B.** -2 **C.** -1 **D.** 0 **E.** 1 **F.** 2 **G.** 3 **H.** 4

2. The dimension of Nul(A), where $A = \begin{bmatrix} 0 & 1 & 2 & -2 & -1 & 3 & 0 \\ 1 & 3 & 1 & 1 & 2 & 0 & 0 \\ -1 & 3 & 4 & 2 & -2 & -1 & 0 \end{bmatrix}$, is given by:

A. 0 **B.** 1 **C.** 2 **D.** 3 **E.** 4 **F.** 5 **G.** 6 **H.** 7

3. It is given that

$$A = \begin{bmatrix} \| & \| & \| & \| & \| \\ \mathbf{a}_1 & \mathbf{a}_2 & \mathbf{a}_3 & \mathbf{a}_4 & \mathbf{a}_5 \\ \| & \| & \| & \| & \| \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Which of the following sets can be taken as a basis for Col A (for any matrix A satisfying the above condition)?

(I) $\{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$

(II) $\{\mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_5\}$

A. None **B.** Only (I)
C. Only (II) **D.** (I) and (II)

4. The solution set of the system $A\mathbf{x} = 0$ has a basis that consists of four vectors and A is a 7×9-matrix. What is the rank of A?

A. 1 **B.** 2 **C.** 3
D. 4 **E.** 5 **F.** 6
G. 7 **H.** There is no sufficient information to determine the rank

5. Suppose that X, Y, Z are 3×3 matrices such that $XYZ = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$. Which of the matrices must be invertible?

A. None **B.** Only X **C.** Only Y **D.** Only Z
E. Only X,Y **F.** Only X,Z **G.** Only Y,Z **H.** X,Y,Z

6. Find the determinant $\det(A)$ of $A = \begin{bmatrix} 2 & 3 & 0 & 0 \\ 2 & 2 & 3 & 0 \\ 2 & 2 & 2 & 3 \\ 2 & 2 & 2 & 2 \end{bmatrix}$:

A. -8 **B.** -6 **C.** -4 **D.** -2 **E.** 0 **F.** 2 **G.** 4 **H.** 6

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For the following two questions, let $A = LU$ be the LU-decomposition of the matrix

$$A = \begin{bmatrix} 3 & -7 & -2 \\ -3 & 5 & 1 \\ 6 & -4 & 0 \end{bmatrix}$$

7. The first column of L is equal to

A. $\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$ **B.** $\begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ **D.** $\begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$ **E.** $\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$ **F.** $\begin{bmatrix} -3 \\ -1 \\ 2 \end{bmatrix}$ **G.** $\begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$ **H.** $\begin{bmatrix} -3 \\ 1 \\ -2 \end{bmatrix}$

8. The last column of U is equal to

A. $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ **B.** $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$ **C.** $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ **E.** $\begin{bmatrix} -2 \\ -1 \\ -\frac{1}{2} \end{bmatrix}$ **F.** $\begin{bmatrix} -1 \\ -2 \\ -1 \end{bmatrix}$ **G.** $\begin{bmatrix} -2 \\ -1 \\ -1 \end{bmatrix}$ **H.** $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

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For the following two questions consider the following basis of $\mathbb{R}^2$:

$$\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \end{bmatrix} \right\}$$

9. Find the coordinate vector $[5\mathbf{e}_2]_{\mathcal{B}}$:

A. $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **B.** $\begin{bmatrix} 0 \\ 5 \end{bmatrix}$ **C.** $\begin{bmatrix} 5 \\ 0 \end{bmatrix}$ **D.** $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ **E.** $\begin{bmatrix} -5 \\ 10 \end{bmatrix}$ **F.** $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ **G.** $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ **H.** $\begin{bmatrix} 10 \\ 5 \end{bmatrix}$

10. The matrix $[T]_{\mathcal{B}}$ of the transformation $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} -x_1 - x_2 \\ 4x_1 + 3x_2 \end{bmatrix}$ relative to $\mathcal{B}$ is given by the matrix:

A. $\begin{bmatrix} 1 & 5 \\ 0 & 1 \end{bmatrix}$ **B.** $\begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$ **C.** $\begin{bmatrix} -1 & -1 \\ 4 & 3 \end{bmatrix}$ **D.** $\begin{bmatrix} 5 & 1 \\ 1 & 0 \end{bmatrix}$

E. $\begin{bmatrix} 0 & 1 \\ 1 & 5 \end{bmatrix}$ **F.** $\begin{bmatrix} -3 & -1 \\ 11 & 2 \end{bmatrix}$ **G.** $\begin{bmatrix} -1 & 4 \\ -1 & 3 \end{bmatrix}$ **H.** $\begin{bmatrix} -3 & 11 \\ -1 & 2 \end{bmatrix}$

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11. For which value of a is 3 an eigenvalue of $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 3 & a \end{bmatrix}$?

A. -3 **B.** -2 **C.** -1 **D.** 0 **E.** 1 **F.** 2 **G.** 3 **H.** 4

12. Which of the following statements are **always** true for square matrices?

(I) If A is upper triangular $\implies A$ is diagonalizable.

(II) If D is a diagonal matrix and $AP = PD \implies A$ is diagonalizable.

A. Both statements are false. **B.** Only (I) is true.
C. Only (II) is true. **D.** Both statements are true.

13. Find a in the matrix A below such that A is diagonalizable:

$$A = \begin{bmatrix} 5 & -2 & 6 & -1 \\ 0 & 3 & a & 0 \\ 0 & 0 & 5 & 4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

A. -8 **B.** -6 **C.** -4 **D.** -2 **E.** 0 **F.** 2 **G.** 4 **H.** 6

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For the following two questions consider the matrix $A = \begin{bmatrix} \sqrt{3} & -1 \\ 1 & \sqrt{3} \end{bmatrix}$.

14. The eigenvalues of A are given by

A. $-1 \pm \sqrt{3}i$ **B.** $1 \pm \sqrt{3}i$ **C.** $\pm 1 + \sqrt{3}i$ **D.** $1, \sqrt{3}$
E. $\sqrt{3} \pm i$ **F.** $1 \pm 3i$ **G.** $\sqrt{3}, \sqrt{3}$ **H.** $\sqrt{3} \pm 3i$

15. The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$, defined by $T(\mathbf{x}) = A\mathbf{x}$ is:

A. A rotation over an angle $\pi/6$ (counter-clockwise), followed by a scaling by factor 2
B. A rotation over an angle $\pi/6$ (counter-clockwise), followed by a scaling by factor 4
C. A rotation over an angle $\pi/3$ (counter-clockwise), followed by a scaling by factor 2
D. A rotation over an angle $\pi/3$ (counter-clockwise), followed by a scaling by factor 4
E. A rotation over an angle $\pi/6$ (clockwise), followed by a scaling by factor 2
F. A rotation over an angle $\pi/6$ (clockwise), followed by a scaling by factor 4
G. A rotation over an angle $\pi/3$ (clockwise), followed by a scaling by factor 2
H. A rotation over an angle $\pi/3$ (clockwise), followed by a scaling by factor 4

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16. Consider the following statements for orthogonal $n \times n$ matrices U and V:

(I) $U + V$ is orthogonal.

(II) UV is orthogonal.

A. Both statements are false. **B.** Only (I) is true.
C. Only (II) is true. **D.** Both statements are true.

17. The distance from $\begin{bmatrix} 1 \\ 5 \\ -10 \end{bmatrix}$ to $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 5 \\ -2 \\ 1 \end{bmatrix} \right\}$ equals:

A. 6 **B.** $3\sqrt{5}$ **C.** 9 **D.** 45
E. 16 **F.** $3\sqrt{14}$ **G.** 126 **H.** 10

18. Applying Gram-Schmidt to the vectors $\mathbf{b}_1 = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 2 \end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \end{bmatrix}$ we obtain, after rescaling, as third vector $\mathbf{v}_3$:

A. $\begin{bmatrix} 1 \\ -7 \\ 0 \\ 4 \end{bmatrix}$ **B.** $\begin{bmatrix} 1 \\ -3 \\ -1 \\ 2 \end{bmatrix}$ **C.** $\begin{bmatrix} 0 \\ -4 \\ 1 \\ 2 \end{bmatrix}$ **D.** $\begin{bmatrix} -1 \\ -1 \\ 2 \\ 0 \end{bmatrix}$ **E.** $\begin{bmatrix} -3 \\ -11 \\ 8 \\ 4 \end{bmatrix}$ **F.** $\begin{bmatrix} 2 \\ 2 \\ 2 \\ 3 \end{bmatrix}$ **G.** $\begin{bmatrix} -5 \\ 3 \\ 8 \\ -4 \end{bmatrix}$ **H.** $\begin{bmatrix} -7 \\ 17 \\ 8 \\ -12 \end{bmatrix}$

19. Determine the least-squares solution of the overdetermined system $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 2 & 1 \\ -2 & 0 \\ 2 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 3 \\ 0 \\ -5 \end{bmatrix}$:

A. $\begin{bmatrix} -1 \\ 2 \end{bmatrix}$ **B.** $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$ **C.** $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ **D.** $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

E. $\begin{bmatrix} -2 \\ -1 \end{bmatrix}$ **F.** $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **G.** $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ **H.** There are no least-squares solutions

END OF PART I.

GO TO PART II: TRUE/FALSE QUESTIONS

Resit *Linear Algebra* (CSE1205): True/False Questions
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- In the following questions you are asked to decide whether the statements are true or false.
- If you think the statement is true, explain clearly why.
- Give a counterexample (with explanation) if you think the statement is false.
- **Simply writing true or false is not enough.**
- Credits: **4 points** for every True/False questions.

20. If A is a 3×3 matrix such that $A\mathbf{x} = 0$ has infinitely many solutions, then $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions for each $\mathbf{b} \in \mathbb{R}^3$.

21. If $\mathbf{v}$ is an eigenvector of the matrices A and B, then $\mathbf{v}$ is also an eigenvector of AB.

22. If the unit vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^4$ are orthogonal, then the vectors $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are also orthogonal.