TI1206M, $16 - 04 - 2018$,   Open Questions

Name: **Student ID:**

*write **readable** and underline your* <u>*surname*</u>

Apart from one definition you are asked to decide whether the statements are **true** or **false**.
Either give a **proof** or a **specific counterexample**.
Note: If a statement is about a general n you cannot give a proof by considering only one value of n.

21. a. Give the definition of linear independence of a set of vectors.

*Write **readable !!!***

21. b. If A is an $m \times n$ matrix with linearly independent columns,
then the null space of A is equal to the set $\{\mathbf{0}\}$.

21. c. If $BC = I$, the identity matrix, for an $m \times n$ matrix B and an $n \times m$ matrix C, then
the columns of C are linearly independent.   (Note: B and C are not necessarily square.)

21. d. Likewise, if $BC = I$, then the columns of B are linearly independent.

*Write **readable !!!***

22. Suppose H is the matrix of the orthogonal projection T onto a subspace W in $\mathbb{R}^n$

a. The column space of H is equal to the orthogonal complement $W^\perp$ of W.

b. $H^2 = H$.

c. $\boxed{3}$ H is orthogonally diagonalizable.   (Start by giving the definition; good for 1 point.)
($Hint$: What are the eigenvalues of H?)

d. $\boxed{1}$ H is a symmetric matrix.