Midterm Test Linear Algebra TI1206M
16 April 2018, 18:30 – 21.30 uur

No calculators are allowed. (**Thinking may preclude long calculations!!**)
Credits: MC questions with 4 alternatives: $\boxed{1}$ point, MC questions with more alternatives: **2** points, questions 21,22: **8** pts. The **final score**: (Total + 4)/5.5, rounded to 1 decimal.
Question 21 and 22 will **only be looked** at if the score for questions 1 – 20 is **at least** 16 points.

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1. Suppose $A = \begin{bmatrix} 2 & 3 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & b \\ c & 2 \end{bmatrix}$. For which values of b and c is $AB = BA$?

a. $b = 2,\ c = 3$ **b.** $b = -2,\ c = 3$ **c.** $b = 3,\ c = -2$ **d.** for all values of b and c
e. $b = 3,\ c = 1$ **f.** $b = 1,\ c = 3$ **g.** $b = 1,\ c = -3$ **h.** for no values of b and c.

2. $\boxed{1}$ Suppose $\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3$ are three vectors in $\mathbb{R}^n$, and let $\mathbf{z}_1 = \mathbf{y}_1 + \mathbf{y}_2$, $\mathbf{z}_2 = \mathbf{y}_2 - \mathbf{y}_3$, and $\mathbf{z}_3 = \mathbf{y}_1 + \mathbf{y}_3$. Mark each of the following statements as true or false:

(I) If $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$ is linearly independent, then so is $\{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3\}$.

(II) If $\{\mathbf{y}_1, \mathbf{y}_2, \mathbf{y}_3\}$ is linearly dependent, then so is $\{\mathbf{z}_1, \mathbf{z}_2, \mathbf{z}_3\}$.

a. Both are true **b.** Only (I) is true **c.** Only (II) is true **d.** Both are false

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Three questions about the matrix $A = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & -1 & 0 & 2 \\ 0 & 1 & 1 & -2 \end{bmatrix}$.

3. Which of the vectors $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ -1 \end{bmatrix}$ and $\mathbf{u}_3 = \begin{bmatrix} 0 \\ 2 \\ 0 \\ 1 \end{bmatrix}$ are in the nullspace of A?

a. none **b.** $\mathbf{u}_1$ **c.** $\mathbf{u}_2$ **d.** $\mathbf{u}_3$ **e.** $\mathbf{u}_1, \mathbf{u}_2$ **f.** $\mathbf{u}_1, \mathbf{u}_3$ **g.** $\mathbf{u}_2, \mathbf{u}_3$ **h.** all three

4. For the same vectors $\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3$, which of the sets $\mathcal{B}_1 = \{\mathbf{u}_1, \mathbf{u}_2\}$, $\mathcal{B}_2 = \{\mathbf{u}_1, \mathbf{u}_3\}$, and $\mathcal{B}_3 = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ can be taken as a basis for Nul A?

a. none **b.** $\mathcal{B}_1$ **c.** $\mathcal{B}_2$ **d.** $\mathcal{B}_3$ **e.** $\mathcal{B}_1, \mathcal{B}_2$ **f.** $\mathcal{B}_1, \mathcal{B}_3$ **g.** $\mathcal{B}_2, \mathcal{B}_3$ **h.** all three

5. Which of the vectors $\mathbf{q}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{q}_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$, and $\mathbf{q}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ are in the column space of A?

a. none **b.** $\mathbf{q}_1$ **c.** $\mathbf{q}_2$ **d.** $\mathbf{q}_3$ **e.** $\mathbf{q}_1, \mathbf{q}_2$ **f.** $\mathbf{q}_1, \mathbf{q}_3$ **g.** $\mathbf{q}_2, \mathbf{q}_3$ **h.** all three

A column space is a subspace

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6. Find the determinant $\begin{vmatrix} 3 & 3 & 3 & 3 \\ 4 & 3 & 3 & 3 \\ 0 & 4 & 3 & 3 \\ 0 & 0 & 4 & 3 \end{vmatrix}$

a. 12 **b.** -12 **c.** 24 **d.** -24 **e.** 18 **f.** -18 **g.** 3 **h.** -3

7. [1] A geometric description of the linear transformation T with standard matrix $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is:

a. a projection **b.** a reflection **c.** a rotation **d.** none of the previous

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Two questions about the LU-decomposition of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ -4 & 0 & 1 \\ 2 & -5 & -4 \end{bmatrix}$

8. The last row of L is equal to

a. $[\,1\ \ -3\ \ 1\,]$ **b.** $[\,-1\ \ 3\ \ 1\,]$ **c.** $[\,2\ \ 3\ \ 1\,]$ **d.** $[\,-2\ \ 3\ \ 1\,]$ **e.** $[\,1\ \ 6\ \ 1\,]$ **f.** $[\,-1\ \ -6\ \ 1\,]$

9. The last column of U is equal to

a. $\begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}$ **c.** $\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ **d.** $\begin{bmatrix} 1 \\ -4 \\ -3 \end{bmatrix}$ **e.** $\begin{bmatrix} 1 \\ 1 \\ -4 \end{bmatrix}$ **f.** $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

Three questions about the matrix $B = PDP^{-1} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}^{-1}$

and the four vectors: $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 2 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

10. The determinant of B is equal to

a. 0 **b.** 1 **c.** -9 **d.** 9 **e.** 11 **f.** 12 **g.** 24 **h.** -36

11. [1] Which of the vectors $\mathbf{v}_1, \mathbf{v}_2$ are eigenvectors of B?

a. both **b.** only $\mathbf{v}_1$ **c.** only $\mathbf{v}_2$ **d.** none

12. [1] Which of the vectors $\mathbf{v}_3, \mathbf{v}_4$ are eigenvectors of B?

a. both **b.** only $\mathbf{v}_3$ **c.** only $\mathbf{v}_4$ **d.** none

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13. For which $c \in \{1,2,3\}$ is $\begin{bmatrix} 1 \\ 2 \\ c \end{bmatrix}$ an eigenvector of the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 6 & -1 & -2 \\ 7 & -2 & -2 \end{bmatrix}$?

a. none **b.** $\{1\}$ **c.** $\{2\}$ **d.** $\{3\}$ **e.** $\{1,2\}$ **f.** $\{1,3\}$ **g.** $\{2,3\}$ **h.** $\{1,2,3\}$

14. The matrix A is given by $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 0 \\ -1 & 1 & 2 \end{bmatrix}$.

Which of the numbers $\lambda_1 = -2,\ \lambda_2 = 2+i,\ \lambda_3 = -2+i$ is/are eigenvalues of A?

a. none **b.** λ_1 **c.** λ_2 **d.** λ_3 **e.** λ_1, λ_2 **f.** λ_1, λ_3 **g.** λ_2, λ_3 **h.** all three

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Two questions about the bases $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\} = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$ and $\mathcal{E} = \{\mathbf{e}_1, \mathbf{e}_2\} = \left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^2$, and the transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with matrix $[T]_{\mathcal{B}} = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}$.

15. Find the coordinate vector $[\mathbf{e}_2]_{\mathcal{B}}$.

a. $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 \\ -3 \end{bmatrix}$ **c.** $\begin{bmatrix} 3 \\ -1 \end{bmatrix}$ **d.** $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$ **e.** $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$ **f.** $\begin{bmatrix} -3 \\ 1 \end{bmatrix}$

16. Find the standard matrix of T. (In other words: Find $[T]_{\mathcal{E}}$.)

a. $\begin{bmatrix} -3 & 7 \\ -1 & 2 \end{bmatrix}$ **b.** $\begin{bmatrix} 3 & -7 \\ 1 & -2 \end{bmatrix}$ **c.** $\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$ **d.** $\begin{bmatrix} -4 & -1 \\ 3 & -2 \end{bmatrix}$ **e.** $\begin{bmatrix} -9 & -3 \\ 7 & 4 \end{bmatrix}$ **f.** $\begin{bmatrix} 9 & 3 \\ -7 & -4 \end{bmatrix}$

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17. The orthogonal projection of the vector $\mathbf{y} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$ onto the line spanned by $\mathbf{a} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$ is given by

a. $\frac{1}{2}\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$ **b.** $\frac{4}{17}\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$ **c.** $\frac{21}{34}\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$ **d.** $\begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix}$ **e.** $\frac{1}{2}\begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix}$ **f.** $\frac{1}{7}\begin{bmatrix} 10 \\ -20 \\ 30 \end{bmatrix}$

Two questions about the set of vectors $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 1 \end{bmatrix} \right\}$

If the Gram-Schmidt construction is applied to this set, yielding $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$:

18. [1] Up to scaling the vector $\mathbf{u}_2$ is equal to

a. $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$ **b.** $\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$ **c.** $\begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix}$ **d.** $\begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

19. Up to scaling the vector $\mathbf{u}_3$ is equal to

a. $\begin{bmatrix} 2 \\ 1 \\ -2 \\ -1 \end{bmatrix}$ **b.** $\begin{bmatrix} 2 \\ -1 \\ -2 \\ 1 \end{bmatrix}$ **c.** $\begin{bmatrix} 2 \\ -5 \\ -2 \\ -1 \end{bmatrix}$ **d.** $\begin{bmatrix} 2 \\ -5 \\ 2 \\ 1 \end{bmatrix}$ **e.** $\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ **f.** $\begin{bmatrix} 1 \\ -1 \\ -2 \\ 1 \end{bmatrix}$

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20. Find the least squares solution $\hat{\mathbf{x}} = \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix}$ of the system of equations

$$\begin{cases} x_1 + 2x_2 = 1 \\ x_1 - x_2 = 2 \\ 2x_1 + x_2 = 6 \end{cases}$$

a. $\begin{bmatrix} 5/3 \\ -1/3 \end{bmatrix}$ **b.** $\begin{bmatrix} 5/3 \\ 1/3 \end{bmatrix}$ **c.** $\begin{bmatrix} 7/3 \\ -2/3 \end{bmatrix}$ **d.** $\begin{bmatrix} 7/3 \\ -1/3 \end{bmatrix}$ **e.** $\begin{bmatrix} 8/3 \\ -1/3 \end{bmatrix}$ **f.** $\begin{bmatrix} 8/3 \\ -2/3 \end{bmatrix}$ **g.** $\begin{bmatrix} 3 \\ -1/3 \end{bmatrix}$ **h.** $\begin{bmatrix} 3 \\ -2/3 \end{bmatrix}$

End of part 1. Now GOTO part 2: Open Questions

Tip: ***If*** *you have time left: Questions 1, 3, 5, 13, 14 and 20 may be the ones most sensitive to computation errors.*

"Answers"

1 e. $b = 3$, $c = 1$

2 c. Only (II) is true

3 h. all three

4 e. $\mathcal{B}_1, \mathcal{B}_2$

5 f. $\mathbf{q}_1, \mathbf{q}_3$

6 h. -3

7 d. none of the previous

8 a. $\begin{bmatrix} 1 & -3 & 1 \end{bmatrix}$

9 f. $\begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$

10 h. -36

11 c. only $\mathbf{v}_2$

12 a. both

13 f. 1 and 3

14 c. λ_2

15 d. $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

16 b. $\begin{bmatrix} 3 & -7 \\ 1 & -2 \end{bmatrix}$

17 e. $\frac{1}{2}\begin{bmatrix} 3 \\ -6 \\ 9 \end{bmatrix}$

18 a. $\begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

19 e. $\begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

20 e. $\begin{bmatrix} 8/3 \\ -1/3 \end{bmatrix}$