Exam Linear Algebra TI1206M (resit)

05 – 07 – 2018, 18:30 – 21.30 h

No calculators are allowed. (**Thinking may preclude long calculations!!**)
Credits: MC questions with 4 alternatives: 1 point, MC questions with more choices: **2** points, questions 23,24: **8 + 9** pts. The **final score**: (Total + 3)/5.5, rounded to 1 decimal.

Multiple Choice Part

1. 1 Suppose $A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$. Which of these matrices is/are reduced echelon matrices?

a. Both **b.** A only **c.** B only **d.** None

2. Find all values of h and k for which the system of equations with augmented matrix $\left[\begin{array}{cc|c} 1 & h & 3 \\ 3 & 6 & k \end{array}\right]$ has a unique solution.

a. $h = 2,\ k = 9$ **b.** $h = 2,\ k \neq 9$ **c.** $h \neq 2,\ k \neq 9$ **d.** $h \neq 2,\ k = 9$
e. $h = 2,\ k \in \mathbb{R}$ **f.** $h \neq 2,\ k \in \mathbb{R}$ **g.** $k = 9,\ h \in \mathbb{R}$ **h.** $k \neq 9,\ h \in \mathbb{R}$

3. 1 Suppose A is a 5×4 matrix and B a 4×3 matrix.
Which of the following statements is/are true:

(I) If $\mathbf{v}$ is in Col(A), then $\mathbf{v}$ is in Col(AB).
(II) If $\mathbf{w}$ is in Nul(B), then $\mathbf{w}$ is in Nul(AB).

a. Both are true **b.** only (I) is true **c.** only (II) is true **d.** Both are false

4. Suppose the equation $(A + B)^T C^T = D$ holds for four invertible $n \times n$ matrices A, B, C, D.
'Solving' for A gives: $A =$

a. $(C^T)^{-1}D^T - B$ **b.** $C^{-1}D^T - B$ **c.** $D^T(C^T)^{-1} - B$ **d.** $D^TC^{-1} - B$
e. $(C^T)^{-1}D^T - B^T$ **f.** $C^{-1}D^T - B^T$ **g.** $D^T(C^T)^{-1} - B^T$ **h.** $D^TC^{-1} - B^T$

Two questions about the three vectors $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 1 \\ r \\ s \end{bmatrix}$.

5. Find all values of r and s for which $\mathbf{w} \in \text{span}\{\mathbf{u}, \mathbf{v}\}$.

a. all r, s **b.** $r = 2, s = 1$ **c.** $r = 3 - s$ **d.** $s = 2r - 3$ **e.** $s = 5 - 2r$ **f.** no r, s

6. Find all values of r and s for which $\mathbf{w} \in (\text{span}\{\mathbf{u}, \mathbf{v}\})^{\perp}$.

a. no r, s **b.** $r = 0,\ s = 1$ **c.** $r = s - 1$ **d.** $r = 1 - s$
e. $r = \frac{1}{3},\ s = \frac{2}{3}$ **f.** $r = -\frac{2}{3},\ s = \frac{1}{3}$ **g.** $r = -\frac{1}{3},\ s = \frac{2}{3}$ **h.** $r = -\frac{2}{3},\ s = -\frac{1}{3}$

7. Find the matrix of the linear transformation $T:\mathbb{R}^2 \to \mathbb{R}^2$ that first rotates points counterclockwise around the origin through $\frac{1}{4}\pi$ radians and then projects (orthogonally) onto the x_2-axis.

a. $\begin{bmatrix} 0 & 0 \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix}$ **b.** $\begin{bmatrix} 0 & 0 \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix}$ **c.** $\begin{bmatrix} 0 & 0 \\ -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix}$ **d.** $\begin{bmatrix} 0 & 0 \\ -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix}$

e. $\begin{bmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix}$ **f.** $\begin{bmatrix} \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix}$ **g.** $\begin{bmatrix} -\frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix}$ **h.** $\begin{bmatrix} -\frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \\ \frac{1}{2}\sqrt{2} & -\frac{1}{2}\sqrt{2} \end{bmatrix}$

8. $\boxed{1}$ It is given that the matrix A has inverse $D = \begin{bmatrix} 8 & 6 & 2 \\ 3 & 10 & 7 \\ 9 & 6 & 5 \end{bmatrix}$. Further, let $B = 3A$, $E = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ and $C = EA$. Mark each of the two identities as true or false:

(I) $B^{-1} = 3D$. (II) $C^{-1} = \begin{bmatrix} 9 & 6 & 5 \\ 3 & 10 & 7 \\ 8 & 6 & 2 \end{bmatrix}$.

a. Both are true **b.** only (I) is true **c.** only (II) is true **d.** both are false

9. Find the determinant of the matrix $F = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$

a. 0 **b.** 2 **c.** -3 **d.** 4 **e.** -5 **f.** 6 **g.** 8 **h.** 12

10. It is given that the 'general' matrix $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$ has determinant equal to 7.

Find the determinants of $B = \begin{bmatrix} a & b & c \\ 2d+g & 2e+h & 2f+i \\ 3g & 3h & 3i \end{bmatrix}$ and $C = \begin{bmatrix} d & e & f \\ g & h & i \\ a & b & c \end{bmatrix}$.

a. $\det B = 7,\ \det C = 7$ **b.** $\det B = 21,\ \det C = 7$ **c.** $\det B = 42,\ \det C = 7$
d. $\det B = 7,\ \det C = -7$ **e.** $\det B = 21,\ \det C = -7$ **f.** $\det B = 42,\ \det C = -7$

Two questions about the matrix $A = \begin{bmatrix} 1 & -1 & 2 & -2 \\ 1 & -1 & 2 & -2 \\ -1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \end{bmatrix}$ and the vectors $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 2 \\ 2 \end{bmatrix}$, $\mathbf{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

11. $\boxed{1}$ True or false: (I) $\mathbf{v} \in \text{Col}(A)$; (II) $\mathbf{v} \in \text{Nul}(A)$.

a. Both are true **b.** only (I) is true **c.** only (II) is true **d.** both are false

12. $\boxed{1}$ True or false: (I) $\mathbf{w}$ is an eigenvector of A; (II) $\mathbf{w} \in (\text{Col}(A))^{\perp}$.

a. Both are true **b.** only (I) is true **c.** only (II) is true **d.** both are false

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13. $\boxed{1}$ Suppose the 4×4 (real) matrix A has the eigenvalues $\lambda_{1,2} = 1 \pm i$ and $\lambda_{3,4} = 3 \pm 2i$. What can you say about the following two statements:

(I) A is (real) diagonalizable. Needs independent eigenvalues

(II) A is symmetric.

a. Both are true **b.** only (I) is true **c.** only (II) is true **d.** both are false

Look up definitions: similar, symmetric & related terms.

14. $\boxed{1}$ Compute $||\mathbf{v}||$ for the vector $\mathbf{v} = \begin{bmatrix} 1 \\ -2 \\ 3 \\ -4 \end{bmatrix}$.

a. 10 **b.** $\sqrt{10}$ **c.** 30 **d.** $\sqrt{30}$

15. $\boxed{1}$ Suppose $\hat{\mathbf{y}}$ is the orthogonal projection of a vector $\mathbf{y}$ in $\mathbb{R}^n$ onto a subspace W, and $\mathbf{w}$ is an arbitrary vector in W. What can you say about the following two statements:

(I) $||\hat{\mathbf{y}}|| \leq ||\mathbf{y}||$

(II) $||\hat{\mathbf{y}} - \mathbf{w}|| \leq ||\mathbf{y} - \mathbf{w}||$

a. Both are true **b.** only (I) is true **c.** only (II) is true **d.** both are false

16. $\boxed{1}$ Which of the matrices $E = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 1 & 0 \end{bmatrix}$ and $F = \begin{bmatrix} 1 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is/are orthogonal?

inverse = normal

a. Both **b.** only E **c.** only F **d.** none

17. Suppose $A = PDP^{-1}$, with $D = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

Then it follows that: $A^{20} =$

a. I **b.** $-I$ **c.** D **d.** $-D$ **e.** A **f.** $-A$

18. The matrix A is given by $A = \begin{bmatrix} 2 & 0 & 4 \\ 2 & -1 & 2 \\ -2 & 0 & 4 \end{bmatrix}$.

Which of the numbers $\lambda_1 = 3 + 3i$, $\lambda_2 = 1$, $\lambda_3 = -4 + 2i$ is/are eigenvalues of A?

a. none **b.** λ_1 **c.** λ_2 **d.** λ_3 **e.** λ_1, λ_2 **f.** λ_1, λ_3 **g.** λ_2, λ_3 **h.** all three

19. The orthogonal projection of the vector $\mathbf{y} = \begin{bmatrix} 3 \\ 5 \\ 27 \end{bmatrix}$ onto the line spanned by $\mathbf{a} = \begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$ is given by

a. $\frac{26}{104}\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$ **b.** $\frac{47}{13}\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$ **c.** $4\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$ **d.** $\frac{7}{2}\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$ **e.** $\frac{45}{26}\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$ **f.** $\frac{105}{26}\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$

20. Find the orthogonal projection of the vector $\mathbf{y} = \begin{bmatrix} 4 \\ 7 \\ 4 \end{bmatrix}$ onto the plane spanned by

$\mathbf{a}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $\mathbf{a}_2 = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$. Note: $\mathbf{a}_1$ and $\mathbf{a}_2$ are not orthogonal.

a. $\begin{bmatrix} 4 \\ 3 \\ 6 \end{bmatrix}$ **b.** $\begin{bmatrix} 4 \\ 2 \\ 4 \end{bmatrix}$ **c.** $\begin{bmatrix} 4 \\ 11 \\ 2 \end{bmatrix}$ **d.** $\begin{bmatrix} -4 \\ 11 \\ 2 \end{bmatrix}$ **e.** $\begin{bmatrix} -4 \\ 11 \\ 4 \end{bmatrix}$ **f.** $\begin{bmatrix} 8 \\ 14 \\ 8 \end{bmatrix}$

21. Suppose $W = \text{span}\{\mathbf{w}_1, \mathbf{w}_2\}$, where the vectors are given by $\mathbf{w}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}$ and $\mathbf{w}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Also, let $\mathbf{b}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 2 \\ -1 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{b}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$.

Which of the sets $\mathcal{B}_1 = \{\mathbf{b}_1, \mathbf{b}_2\}$, $\mathcal{B}_2 = \{\mathbf{b}_1, \mathbf{b}_3\}$ and $\mathcal{B}_3 = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$ give bases for the orthogonal complement of W?

a. none **b.** $\mathcal{B}_1$ **c.** $\mathcal{B}_2$ **d.** $\mathcal{B}_3$ **e.** $\mathcal{B}_1, \mathcal{B}_2$ **f.** $\mathcal{B}_1, \mathcal{B}_3$ **g.** $\mathcal{B}_2, \mathcal{B}_3$ **h.** all three

22. It is given that the (symmetric!) matrix $A = \begin{bmatrix} 6 & -2 \\ -2 & 9 \end{bmatrix}$ has the eigenvalues $\lambda_1 = 5$ and $\lambda_2 = 10$. An <u>orthogonal</u> matrix that diagonalizes A is given by

a. $\frac{1}{2}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ **b.** $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$ **c.** $\frac{1}{5}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$ **d.** $\frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$

e. $\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$ **f.** $\frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ -1 & 2 \end{bmatrix}$

End of part 1. Now GOTO part 2: Open Questions

"Answers" (The red questions are worth 1 point.)

1 b. A only

2 f. $h \neq 2,\ k \in \mathbb{R}$

3 c. (II) only

4 b. $C^{-1}D^T - B$

5 d. $s = 2r - 3$

6 f. $r = -\frac{2}{3},\ s = \frac{1}{3}$

7 a. $\begin{bmatrix} 0 & 0 \\ \frac{1}{2}\sqrt{2} & \frac{1}{2}\sqrt{2} \end{bmatrix}$

8 d. Both are false

9 g. 8

10 c. Both are true $\det B = 42,\ \det C = 7$

11 a. Both are true

12 b. only (I) is true

13 d. Both are false.

14 d. $\sqrt{30}$ Quicky!

15 a. Both are true

16 d. none

17 f.

18 a. none

19 c. $4\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$

20 a. $\begin{bmatrix} 4 \\ 3 \\ 6 \end{bmatrix}$

21 b. only $\mathcal{B}_1$

22 d. $\frac{1}{\sqrt{5}}\begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix}$