Exam Complex Function Theory AM2040

Monday June 29, 2020, 13:30–16:30

TU Delft

- Open book exam: You are allowed to use the book and your notes. You are not allowed to use the internet or help from other people.
- Every answer must be motivated by a calculation, a logical argumentation or a reference to the theory. You cannot refer to results of exercises.
- In the final answer write a complex number in the form $a + bi$.
- The points for each question are in the margin. There are 90 points in total.
- The course is evaluated with a Pass or Fail. A Pass requires an average grade of at least 5.8. (Midterm 10%, exam 90%).
- Your work must be hand-written. Use a separate piece of paper for each question (except for question 1). Upload your handwritten work as **1 pdf file**.
- When scanning your work, place your student-ID on the first page.
- In case of a technical problem email w.g.m.groenevelt@tudelft.nl as soon as possible. When possible attach the images of your work in this email.

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(0) 1. Write the following sentence on your exam:

> I declare that I have made this examination on my own, with no assistance and in accordance with the TU Delft policies on plagiarism, cheating and fraud.

(10) 2. Let the function L be a branch of the logarithm with branch cut

$$\{z \in \mathbb{C} \mid \mathrm{Re}(z) = 0,\ \mathrm{Im}(z) \geq 0\}.$$

Evaluate

$$\int_{[iR,-1+iR,-i,1+iR,iR]} L(z)\,dz, \qquad R > 0.$$

(10) 3. Determine and classify the isolated singularities of $f(z) = \dfrac{z + i\pi}{z\sinh^2(z)}$.

(10) 4. Show that $g(z) = \sum_{n=1}^{\infty} \frac{z^n}{1+z^n}$ is analytic on $\{z \in \mathbb{C} \mid \ |z| < 1\}$.

(12) 5. Determine the Laurent series expansion of

$$h(z) = \frac{i}{(z+i)(z-3i)}$$

on the region $\{z \in \mathbb{C} \mid 1 < |z-2i| < 3\}$.

6. Let f be an entire function.

(10) a) Prove that $f^*(z) := \overline{f(\overline{z})}$ is also entire.

(10) b) Suppose $f(z) \in \mathbb{R}$ for $z \in (-2020, 2020)$. Show that

$$f(\overline{z}) = \overline{f(z)}, \qquad z \in \mathbb{C}.$$

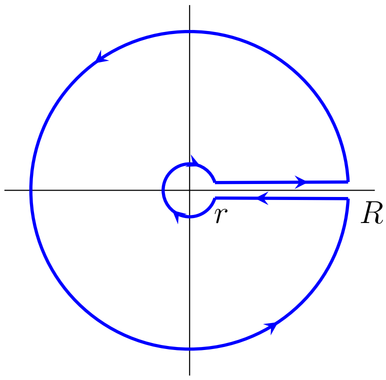
(12) 7. Let $a \in \mathbb{R}$ with $|a| > 1$. Evaluate

$$\int_0^{2\pi} \frac{1}{1+a^2-2a\cos(\theta)}\, d\theta.$$

(16) 8. Evaluate

$$\int_0^{\infty} \frac{x^{\frac{1}{3}}}{x^2+x+1}\, dx$$

using the path as indicated in the picture.



Examiner: W. Groenevelt
Exam reviewer: J. de Groot