Exam Complex Function Theory AM2040

Friday July 24, 2020, 13:30–16:30

- Open book exam: You are allowed to use the book and your notes. You are not allowed to use the internet or help from other people.
- Every answer must be motivated by a calculation, a logical argumentation or a reference to the theory. You cannot refer to results of exercises.
- In the final answer write a complex number in the form $a + bi$.
- The points for each question are in the margin. There are 90 points in total.
- The course is evaluated with a Pass or Fail. A Pass requires at least 48 points.
- Your work must be hand-written. Use a separate piece of paper for each question (except for question 1). Upload your handwritten work as **1 pdf file**.
- When scanning your work, place your student-ID on the first page.
- In case of a technical problem email w.g.m.groenevelt@tudelft.nl as soon as possible. When possible attach the images of your work in this email.

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(0) 1. Write the following sentence on your exam:

> I declare that I have made this examination on my own, with no assistance and in accordance with the TU Delft policies on plagiarism, cheating and fraud.

(10) 2. Let $z^{\frac{1}{2}}$ be the square root function with branch cut $\{z \in \mathbb{C} \mid \mathrm{Im}(z) = 0,\ \mathrm{Re}(z) \geq 0\}$. Define

$$F(z) = \left(\frac{1+z}{1-z}\right)^{\frac{1}{2}}.$$

Find the largest region on which F is analytic.

(10) 3. Prove or disprove: There exists an analytic function $f : \mathbb{C} \to \mathbb{C}$ with

$$\mathrm{Re}(f(x+iy)) = x^3 y.$$

(10) 4. Let g be an entire function satisfying

$$|g(z)| \leq \ln(|z|+1), \qquad z \in \mathbb{C}.$$

Show that g is constant.

(15) 5. Determine and classify the isolated singularities in $\mathbb{C}$ (not ∞) of

$$h(z) = \frac{\cos(\frac{i\pi}{z}) + 1}{z^2+1}.$$

6. Determine the Laurent series expansion of the following functions on the region $\{z \in \mathbb{C} : |z+i| > 4\}$.

(5) a) $\dfrac{1}{iz+3}$

(5) b) $\dfrac{1}{(iz+3)^2}$

(15) 7. Evaluate the improper integral

$$\int_{-\infty}^{\infty} \frac{\sin(x)}{x^2+2x+2}\,dx.$$

(10) 8. a) Show that $z^{2020}+z+1$ has all its zeros in $\{z \in \mathbb{C} : |z| < 2\}$.

(10) b) Evaluate

$$\int_{C_2(0)} \frac{z^{2019}}{z^{2020}+z+1}\,dz,$$

where $C_2(0)$ has positive orientation.
Hint: consider the integral over $C_R(0)$ with $R > 2$ and use part a.

Examiner: W. Groenevelt
Exam reviewer: J. de Groot