

Final exam Ordinary Differential Equations, AM2030
Monday 25 January 2021, 13.30-16.30h

- The exam is open-book: you may use the book, your own notes, and the table of Laplace transforms provided. You are also allowed to use a graphical calculator. You are not allowed to use any other external sources of information, including, but not limited to, Maple, any other resources on Brightspace or the wider Internet, or other people.
- Please follow the **instructions given on Brightspace** when handing in your work. Clearly write your last name, first name, and student number at the top of each page you hand in. Create a separate pdf (no other file type) for each of the five questions and use the following naming convention for your files:
  lastname_firstname_studentnumber_exercisenumber.pdf.
  For example: doe_john_5151515_1.pdf or doe_jane_5353535_2.pdf.
- As the **first two sentences of the exam** please write and adhere to the following statement:
  "I declare that I have made this examination on my own, with no unauthorized assistance from people or other sources and in accordance with the TU Delft policies on plagiarism, cheating and fraud. I created the submitted answers all by myself during the time slot that was allocated for that specific exam part."
- Unless explicitly stated otherwise, you are required to provide clear proofs for any statements you make. In particular, if you use a result from the book or reader, show that all the required assumptions hold, and clearly state which conclusion(s) you draw.
- You may write your answers in Dutch or English.
- You cannot discuss the questions with anyone before Monday February 1, 2021.
- If you participate in this exam, you may be selected afterwards for an online remote face-to-face check. If you are selected, participation in this check will be required to validate your result on the exam.
- This exam has 3 questions. The grade is $(10 \cdot \#\text{marks})/47$, rounded to tenths.

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1 $$w''(t) + 2w'(t) + 3w(t) + 3 = 0, \qquad w(0) = 0, \quad w'(0) = -3 \tag{1}$$

(a) Solve the initial value problem in (1). *[6 pts]*

(b) Rewrite (1) into an initial value problem for a system of first order ODEs. *[3 pts]*

(c) Use the Laplace transform to solve the system from (b) and check that your answer is consistent with your answer in part (a). *[6 pts]*

2 (a) Compute e^{At}, with $A = \begin{pmatrix} 2 & 5 \\ 0 & 2 \end{pmatrix}$. *[8 pts]*

(b) Solve the initial value problem

$$x'(t) = Ax(t) + \begin{pmatrix} 2\cos(4t) \\ 0 \end{pmatrix}, \qquad x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}.$$

[6 pts]

3 $$\begin{aligned} x'(t) &= x(t)\big(4 - 2y(t)\big), \\ y'(t) &= y(t)\big(-3 + 3x(t)\big). \end{aligned} \tag{2}$$

(a) Determine both equilibrium points of the system (2). *[2 pts]*

(b) For each of the equilibrium points, determine the *linear* system that results from linearising (2) around the equilibrium point. *[4 pts]*

(c) For each of the two linear systems from part (b), determine what kind of equilibrium point the origin is: (un)stable node —singular, degenerate, or neither— (un)stable focus, saddle point, or centre? *[4 pts]*

(d) For each of the two linear systems from part (b), sketch the phase portrait. Explicitly compute and indicate in your sketch the important directions determined by the eigenvectors associated with the systems. *[6 pts]*

(e) Choose one of the equilibrium points of the *nonlinear* system (2) from part (a) and determine if it is stable, asymptotically stable, or unstable. *[2 pts]*

This is the end of the exam.