Delft University of Technology
Faculty EWI

Exam WI 4052, Risk Analysis
19.01.2021

This is an open book exam. The exam consists of 5 questions.
Write your solutions to each question on separate pages and submit them via Brightspace submission system (as you did with assignments) in pdf format, each named YourStudentNumber_Question_x, x=1,2,3,4,5. Show all your computations, otherwise your solutions will look suspicious if it is not visible how you arrive with the result. In such cases I will not give you any points.
Each exercise is worth 5 points. The information about how these points are assigned is shown in each subpart of the question (xp).

On each page of the exam solutions write the following declaration:

"I declare that I made this exam on my own, without assistance and in accordance to TUD policy on plagiarism, cheating and fraud".

1. Let $Y_1, \dots, Y_k$ denote lifetimes of k mutually independent components of a system. Assume that Y_i is exponential with parameter λ_i, i=1,…,k.

   a) (3p) Compute the probability that the lifetime of i^{th} component is the smallest out of $Y_1, \dots, Y_k$ (*show all computations*).
   b) (2p) What is the probability that the i^{th} component fails as first before time t?

2. Consider a study following 10 early breast cancer patients at a hospital over 10 months. Each patient had clinic visits to be examined for cosmetic appearance such as breast retraction. However, actual visit times differ from patient to patient. Among the 10 patients, 2 did not experience breast retraction during the study (no retraction during 10 months). There are 3 patients for whom the breast retraction happened before the first clinical visit which took place in month 2. For other patients, intervals such as (5,7], (2,4], (3,4], (3,5] and (2,7] were observed for their breast retraction times (Here the interval (x,y] means that the patient had a clinic visit at month x and no breast retraction was detected at the visit, while at the next visit at month y, breast retraction was found to be present already).

   a) (2p) Consider X to be a random variable describing the time of breast retraction of a cancer patient. Denote as F_θ ($\theta>0$) the cumulative distribution function (cdf) of X. Write the likelihood function of the parameter θ given the data described above (*write the likelihood in terms of F_θ*).

   b) (3p) Find the maximum likelihood estimate of θ assuming that the density function of X is a linear function with slope equal to θ, on the interval

(0,A) (*note that A depends on θ, you have to show what A is equal to*) which is equal to zero at 0.

3. X is a random variable with geometric distribution with parameter p. Assume that uncertainty of the parameter p is described as a piecewise constant density on (0,1) such that on the interval $(0,\theta)$, $0<\theta<1$ the density takes a constant value and its value doubles on the interval $(\theta,1)$.
   a) (2.5p) Compute the predictive distribution of X.
   b) (2.5p) Compute the posterior distribution of p after observing that X=2.

   Note: *Show your computations. Providing only formulas without solutions will not get you any points in an open book exam.*

4. Consider a situation where we want to study how strong the immune response to COVID 19 a random person in a population possesses. Two important causes of the immune response can be taken into account: whether a person was infected I -{yes, no} and whether this person was vaccinated V -{yes, no}. It is assumed that 30% of the population have been infected (recently and still have immune response, others who were not infected or were infected long ago belong to the group of not infected). When somebody was recently infected then they do not need to be vaccinated, but also only 40% of people who were not infected have been vaccinated up to the time where the study is curried out. We assume that people who were not vaccinated and were not infected have no immune response, denoted as IR=no. IR will be weak if an infected person had no or mild symptoms (denoted by S) and strong if S were severe. If vaccinated it has been observed that a person will have 55% chance to develop strong IR and 45% weak IR. Also it is known that not infected person will have no symptoms and in case this person is infected then with 50% chance will have no symptoms, 40% mild and 10 % severe.
   a) (2p) Present a clear notation and description of random variables in the story above (which values these random variables can take). Draw a Bayesian belief network that describes dependencies between these variables.
   b) (1p) Which (conditional) independencies are present in this bbn?
   c) (1p) Present the conditional probability tables (cpts) of this bbn? Use the way of specifying cpts as presented during the course (table form).
   d) (1p) Compute P(IR=strong). (*Show all computations*).

5. (5p) Consider a partitioned random vector (X,Y,Z_1, Z_2). Show that the following assertions are equivalent:
   a) Y is independent of (Z_1, Z_2) given X,
   b) Y is independent of Z_2 given (X, Z_1) and Y is independent of Z_1 given X.

   Note: *Include explanations of steps you take in the proof.*