DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

Exam TI2316
Automata, Languages & Computability

July 4, 2017, 13:30–16:30

- If your grade on the midterm was ≥ 5, you **do not have to do questions 1 through 3**. That means you do have to score at least 10 points on questions 4 through 6 (corresponding to a grade of 4 over only those questions), with the resulting final grade being the average of the two parts. You can always do the questions of the first part as well if you are not satisfied with your result on the midterm; the best grade counts.
- **Use separate answer sheets for the 2 parts.**
- Total number of pages (without this cover page): 4.
- This exam consists of 6 open questions of equal weight.
- Consulting handouts, readers, notes, books or other sources during this exam is prohibited. The use of electronic devices such as calculators, mobile phones etc is also prohibited.
- A single exam cannot cover all topics, so do not draw conclusions based on this exam about topics that are never tested.
- Formulate your answers in correct English or Dutch and write legibly (use scrap paper first). Do not give irrelevant information, this could lead to a deduction of points.
- Before handing in your answers, ensure that your name and student number is on every page and indicate the number of pages handed in on (at least) the first page.
- **Note:** for some exercises a maximum is stated for the number of lines an answer can consist of! Exceeding this number will lead to deduction of points.

The following 3 questions are about the first half of the course.

1. Suppose we have the following language L over the alphabet $\Sigma = \{a, b, c\}$:

   $$L = \{w \in \Sigma^* \mid c_a(w) \text{ is even and } w \text{ contains a } b\},$$

   with $c_x(w)$ representing the number of occurrences of the symbol x in w.

   (a) (6 points) Give a deterministic finite automaton (DFA) D with *at most 7 states* such that $L(D) = L$. A transition diagram suffices. Also give a brief description of how D works. *(max. 10 lines)*

   (b) (2 points) Is L context free? Motivate your answer. *(max. 5 lines)*

   (c) (2 points) Suppose we append a string w' to every word of L. Is the resulting language regular? Motivate your answer. *(max. 5 lines)*

2. Suppose we have the following regular expression:

   $$R = ((a \cup b)c^*)^*$$

   (a) (2 points) Give two different words in $L(R)$.

   (b) (5 points) Create an NFA N such that $L(N) = L(R)$. Use no more than 10 states.

   (c) (3 points) Is it possible to create a finite automaton M with $L(M) = \overline{L(R)}$? Motivate your answer. *(max. 10 lines)*

3. Suppose we have languages L_1 and L_2, both defined over alphabet $\Sigma = \{a, b\}$:

   $$L_1 = \{vw \mid v, w \in \Sigma^*,\ v = w^{\mathcal{R}}\}$$

   $$L_2 = \{a^n \mid n \geq 0\}$$

   (a) (2 points) Give a context-free grammar G_1 that generates L_1, i.e., such that $L(G_1) = L_1$.

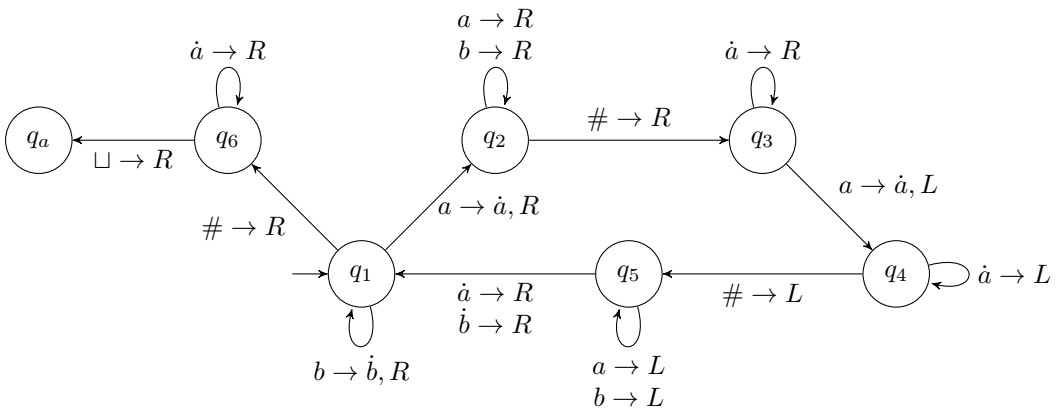
   (b) (3 points) Give a context-free grammar G_2 that generates $\overline{L_2}$, i.e., such that $L(G_2) = \overline{L_2}$.

   (c) (2 points) Give a context-free grammar G_3 such that $L(G_3) = L_1 \cup \overline{L_2}$.

   (d) (3 points) Is G_3 in Chomsky normal form (CNF)? If so, indicate the properties it has that make it so; if not, indicate the properties it has that violate CNF. Mention 3 properties.

The following 3 questions are about the second half of the course.

4. Suppose we have the following transition diagram of Turing machine M with input alphabet $\Sigma = \{a, b, \#\}$ and tape alphabet $\Gamma = \{a, \dot{a}, b, \dot{b}, \#, \sqcup\}$. The tape is bounded on the left, q_a is the accepting state and any undefined transition implicitly leads to the reject state q_r, with the head moving right.



   (a) (2 points) Give the series of configurations that M goes through when processing input $ba\#a$.

(b) (1 point) Does M accept input $ba\#a$? Motivate your answer. *(max. 3 lines)*

(c) (4 points) Give $L(M)$ as a set and explain how you reached your answer. *(max. 20 lines)*

(d) (3 points) Is M a decider for $L(M)$? Indicate how you can see this. *(max. 10 lines)*

5. (a) (3 points) Suppose we have languages L_1, L_2 and L_3, with $L_1 \subseteq L_2 \subseteq L_3$. We also know that L_1 and L_3 are Turing-decidable. Does L_2 have to be decidable as well? Motivate your answer. *(max. 10 lines)*

(b) (3 points) Suppose we have languages L_1, L_2 and L_3, with $L_1 \leq_m L_2$ and $L_2 \leq_m L_3$. We also know that L_1 is Turing-decidable and L_3 is Turing-recognizable. Is L_2 decidable and/or recognizable? Motivate your answer. *(max. 10 lines)*

(c) (4 points) Let E_{PDA} be defined like E_{TM}:

$$E_{\text{PDA}} = \{\langle P \rangle \mid P \text{ is a PDA and } L(P) = \emptyset\}$$

Is E_{PDA} decidable? Motivate your answer. *(max. 20 lines)*

6. Suppose we have the following language:

$$L = \{\langle M, v, w \rangle \mid M \text{ outputs } w \text{ on the tape when run on input } v\}$$

Use mapping reduction to prove that L is undecidable. Give:

(a) (1 point) A suitable problem to reduce from.

(b) (3 points) A suitable reduction function $f : \Sigma^* \to \Sigma^*$.

(c) (6 points) A proof showing f satisfies the requirements of a mapping reduction.