DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TU Delft

Midterm TI2316
Automata, Languages & Computability

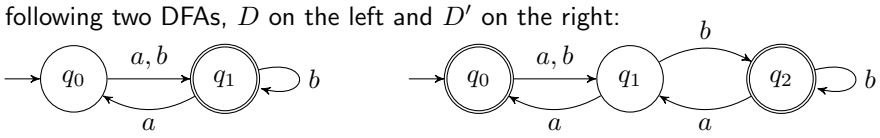
May 22, 2019, 13:30–15:30

- Total number of pages (without this cover page): 1.
- This exam consists of 7 open questions, the weight of each subquestion is indicated on the exam.
- Consulting handouts, readers, notes, books or other sources during this exam is prohibited. The use of electronic devices such as calculators, mobile phones etc is also prohibited.
- A single exam cannot cover all topics, so do not draw conclusions based on this exam about topics that are never tested.
- Formulate your answers in correct English and write legibly (use scrap paper first). Do not give irrelevant information, this could lead to a deduction of points.
- Before handing in your answers, ensure that your name and student number is on every sheet of paper.

Question:	1	2	3	4	5	6	7	Total
Points:	6	6	5	2	4	3	6	32

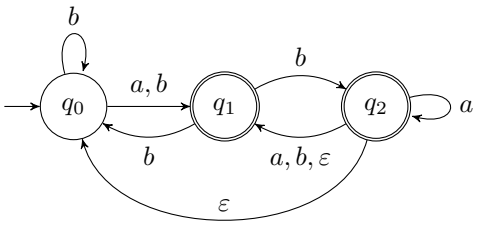
1. Consider a DFA $D = (Q, \Sigma, \delta, q_0, F)$.
   (a) What, if anything, can we conclude about the language $L(D)$ when
      i. (1 point) $|F| = |Q|$? *(Explain your answer in at most 3 lines.)*
      ii. (1 point) $0 < |F| < |Q|$? *(Explain your answer in at most 3 lines.)*
   (b) (2 points) A student claims that if $\delta^*(q_0, \text{pho}) = q_1$, $\delta(q_1, \text{e}) = q_0$, and $\delta^*(q_0, \text{phoenix}) \notin F$, it must also hold that nix $\notin L(D)$. You may assume $\{\text{p,h,o,e,n,i,x}\} \subseteq \Sigma$. Is that reasoning correct? *(Explain your answer in at most 5 lines.)*
   (c) (2 points) Consider now a DFA $D' = (Q, \Sigma, \delta', q_0, F)$. A student claims that since the set of accepting states of D' and D are the same, it must hold that $L(D') = L(D)$. Is the reasoning correct? If so, prove it. If not, provide a counterexample in the form of two DFAs of at most 3 states each.

2. Consider the following two DFAs, D on the left and D' on the right:



   (a) (1 point) Construct a new DFA D'' of at most 2 states, such that $L(D'') = \overline{L(D)}$. A transition diagram and a short explanation suffice.
   (b) (5 points) Use the procedure from Sipser to construct a DFA D''', such that $L(D''') = L(D) \cup L(D')$. You should leave out unreachable states.

3. Consider the NFA N depicted as:



   (a) (1 point) Give a word of length 5 that is in the language $L(N)$.
   (b) (4 points) Add at most 5 states to N and modify the accepting states to create a new NFA N' such that $L(N') = L(N) \circ L(b^*(a \cup ab)^*)$.

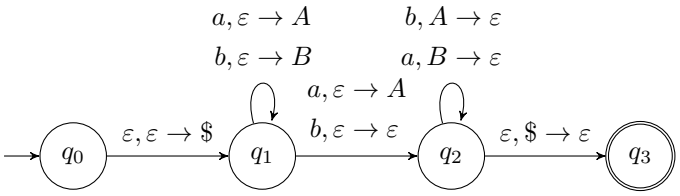
4. (2 points) State three properties a GNFA should adhere to in one line each.

5. Consider the following rules of a CFG $G = (V, \Sigma, R, S)$, over $\Sigma = \{a, b, c\}$:

   $S \to bAb \mid aBa \mid A \mid B$
   $A \to a \mid b$
   $B \to c \mid AbA \mid \varepsilon$

   (a) (1 point) Give a valid set V.
   (b) (3 points) Show that this grammar is ambiguous. *(Explain your answer in at most 6 lines.)*

6. (3 points) Consider the following PDA P; give a grammar G with at most 6 rules such that $L(G) = L(P)$.



7. (6 points) Consider the following language, where the alphabet $\Sigma = \{a, b\}$:

   $$L = \{b^n abv \mid v \in \Sigma^*, n \geq 1, \text{ and } |v| = 8n + 4\}$$

   This language is **not** regular. Prove this using the pumping lemma in at most 20 lines.

End of the exam