DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TU Delft

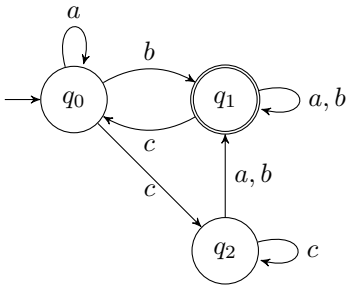
Resit TI2316
Automata, Languages & Computability

August 14, 2018, 9:00–12:00

- Total number of pages (without this cover page): 3.
- This exam consists of 9 open questions, the weight of each subquestion is indicated on the exam.
- Consulting handouts, readers, notes, books or other sources during this exam is prohibited. The use of electronic devices such as calculators, mobile phones etc is also prohibited.
- A single exam cannot cover all topics, so do not draw conclusions based on this exam about topics that are never tested.
- Formulate your answers in correct English or Dutch and write legibly (use scrap paper first). Do not give irrelevant information, this could lead to a deduction of points.
- Before handing in your answers, ensure that your name and student number is on every page and indicate the number of pages handed in on (at least) the first page.
- **Note:** for some exercises a maximum is stated for the number of lines an answer can consist of! Exceeding this number may lead to deduction of points.

Question:	1	2	3	4	5	6	7	8	9	Total
Points:	9	12	6	6	11	7	7	6	6	70

1. Consider the following transition diagram of a DFA D over the alphabet $\Sigma = \{a, b, c\}$:



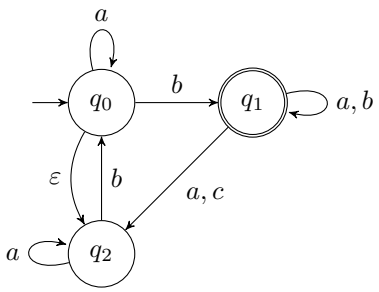
   (a) (1 point) Give a word of length 6 that is in the language $L(D)$.

   (b) (1 point) Give a word that is in the language $\{w | w = uv, u \in \Sigma, v \in \Sigma^* \text{ and } \delta(q_0, u) \in F\}$, where F is the set of accepting states of D.

   (c) (4 points) Construct a DFA D' with at most 4 states over the alphabet $\Sigma = \{0, 1, 2, 3\}$ that accepts all strings that represent even numbers. E.g., $0 \in L(D')$, $00 \in L(D')$, $22 \in L(D')$, but $31 \notin L(D')$.

   (d) (3 points) Describe how to create an NFA N over alphabet $\Sigma = \{a, b, c, 0, 1, 2, 3\}$ such that $L(N) = \overline{L(D)} \cup \overline{L(D')}$.

2. Consider the following NFA N over the alphabet $\Sigma = \{a, b, c\}$:



   (a) (2 points) Take a new NFA N' that is identical to N except that $\delta(q_1, c) = \{q_0\}$. Give a word w, such that $w \notin L(N)$, but $w \in L(N')$.

   (b) (5 points) Using the procedure from Sipser, convert N to a DFA D, s.t. $L(D) = L(N)$. You may leave out unreachable states.

   (c) (2 points) Create a new NFA N'' such that $L(N'') = L(R)$ with $R = (a \cup b)^* c (b \cup a^*)$. Use at most 5 states in your NFA.

   (d) (3 points) Describe how to create a new NFA M, such that

$$L(M) = \{w^2 u v^3 \mid w \in L(D), u \in L(R), v \in L(N)\},$$

   where D is your DFA from question b and R is the regular expression from question c. *(Answer in at most 8 lines.)*

3. Consider the grammar $G = \langle V, \Sigma, R, S \rangle$, with $\Sigma = \{a, b, c\}$ and R:

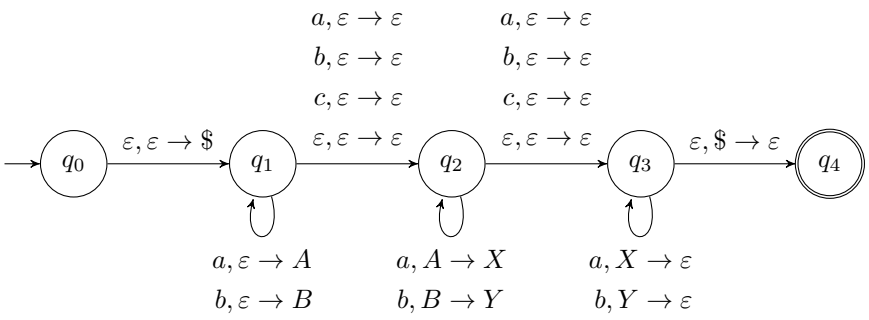
$$S \to aAa \mid bBb \mid cCc$$
$$A \to S \mid aBa$$
$$B \to S \mid bCb$$
$$C \to S \mid \varepsilon \mid a \mid b \mid c$$

   (a) (1 point) Give a valid V for the grammar G.

   (b) (3 points) Is G ambiguous? Motivate your answer. *(Answer in at most 10 lines.)*

   (c) (2 points) Give a rule set R' of a grammar G' such that $L(G') = (L(G) \cup \overline{L(G)})^*$ and $|R'| \leq 6$. Explain how your rule set is constructed in at most 5 lines.

4. Consider the following PDA P over the alphabet $\Sigma = \{a, b, c\}$:



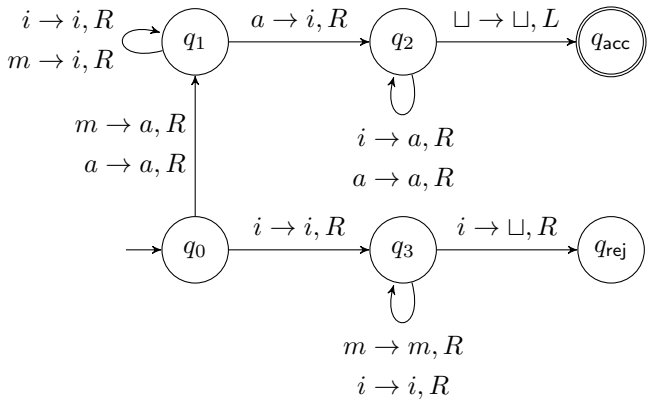
   (a) (3 points) Describe the language of $L(P)$ in set notation. Explain your answer in at most 5 lines.

   (b) (3 points) Consider now the grammar G, with the rule set:

$$S \to AXAXA \mid BXBXB \mid \varepsilon$$
$$A \to aA \mid \varepsilon$$
$$B \to bB \mid \varepsilon$$
$$X \to a \mid b \mid c \mid \varepsilon$$

   Show that $L(G) \neq L(P)$.

5. Consider the Turing Machine T:



   (a) (2 points) The machine is currently in the configuration: $aiiq_1iaiia\sqcup$. What do we know about the initial word on the tape? *(Answer in at most 5 lines.)*

   (b) (2 points) Consider again the configuration from question a. Does the machine end up in an accepting state?

   (c) (2 points) Is T deterministic or non-deterministic? Explain your answer in at most 5 lines.

   (d) (3 points) Is $L(T)$ regular? If so show it, if not explain why not. *(Answer in at most 5 lines.)*

   (e) (2 points) Given an arbitrary NTM N that decides $L(N)$, explain how a DTM D can decide $L(N)$. *(Answer in at most 10 lines.)*

6. Consider the language:

$$L = \{w | w = f(x) \quad \forall x(\text{where } x \text{ is an integer larger than or equal to } 0)\}$$

   where $f(x) = x + 8$.

   (a) (5 points) Give a high-level description of a TM M that enumerates L.

   (b) (2 points) Consider a language L that can be enumerated. Now we also create another machine that recognises $\overline{L}$. What can we conclude about the decidability of L?

7. (a) (3 points) Consider the set A that contains all integers divisible by 42.[1] Show that A is countable.
   (b) (4 points) Consider the following two subsets of $\mathbb{R}$ and explain whether they are countable or not. Answer in at most 5 lines per subset.
      I. $\{x \in \mathbb{R} \mid x = \pi^k, \text{ with } k \in \mathbb{Q}\}$
      II. $\{x \in \mathbb{R} \mid 0 \leq x \leq 0.1\}$.

8. (a) (2 points) Is A_{TM} Turing-recognisable and/or co-Turing-recognisable? Explain your answer. *(Answer in at most 5 lines.)*
   (b) (4 points) Consider now the following problem:

   $$\{M \mid M \text{ is a Turing Machine with 5 states}\}$$

   Show that this problem is co-Turing-recognisable.

9. (6 points) Consider the following problem X:

   $$\{M \mid M \text{ is a Turing Machine, such that } P \subseteq L(M)\}$$

   where P is the set of all prime numbers. Use Rice's theorem to show that X is undecidable.

[1] We say a number is divisible by 42 if after dividing by 42 the result is an integer. I.e., 84 is divisible by 42, but 21 is not.

End of the exam