DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TU Delft

Exam TI2316
Automata, Languages & Computability

July 1, 2019, 9:00–11:00

- Total number of pages (without this cover page): 2.
- This exam consists of 7 open questions, the weight of each subquestion is indicated on the exam.
- Consulting handouts, readers, notes, books or other sources during this exam is prohibited. The use of electronic devices such as calculators, mobile phones etc is also prohibited.
- A single exam cannot cover all topics, so do not draw conclusions based on this exam about topics that are never tested.
- Formulate your answers in correct English and write legibly (use scrap paper first). Do not give irrelevant information, this could lead to a deduction of points.
- Before handing in your answers, ensure that your name and student number is on every page and indicate the number of pages handed in on (at least) the first page.
- **Note:** for some exercises a maximum is stated for the number of lines an answer can consist of! Exceeding this number may lead to deduction of points.

Question:	1	2	3	4	5	6	7	Total
Points:	7	6	6	4	4	5	5	37

1. Consider a Deterministic Turing Machine (DTM) $D = (Q, \Sigma, \Gamma, \delta, q_0, q_{\text{accept}}, q_{\text{reject}})$. You should consider each of the following questions individually (unless otherwise specified), i.e., if changes are made to the machine they are only for that one subquestion.

   (a) (1 point) Someone argues that if the only incoming transition to q_{accept} is $\delta(q_2, a) = (q_{\text{accept}}, c, R)$ then all words that are accepted by the machine must end in an a. Do you agree with this argument? If you agree, explain why. If not, describe a machine D that forms a counterexample.

   (b) (2 points) Consider now an extension of the DTM model in which we change q_{accept} and q_{reject} by a set of accepting states A and a set of rejecting states R respectively. All other properties of the TM remain the same. We call such a deterministic TM an EDTM. Does this change expressivity of the model? That is, are there languages that cannot be recognised by a DTM, that can be recognised by an EDTM? If so, give an example of such a language or a set of requirements it should fulfil and explain them. If not, show how we can construct a classical DTM from such an EDTM.

   (c) Consider now that $\Sigma = \{a\}$, $\Gamma = \Sigma \cup \{\sqcup\}$, $Q = \{q_0, q_1, q_{\text{accept}}, q_{\text{reject}}\}$ and δ specified as follows:

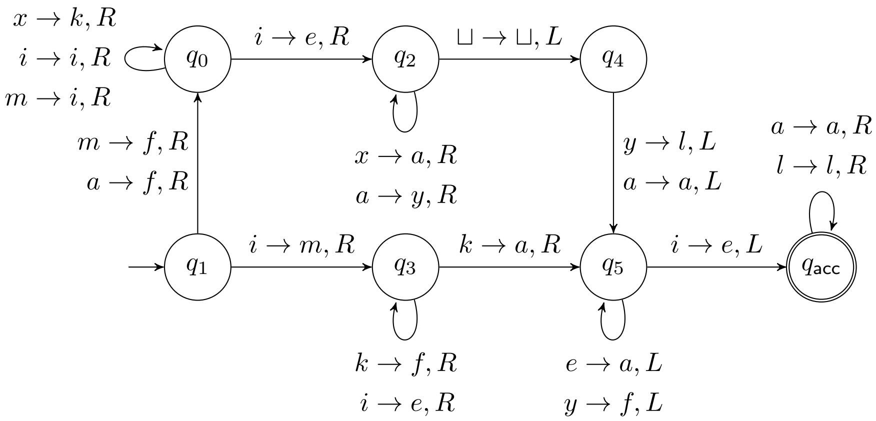
   - $\delta(q_0, a) = (q_1, a, L)$
   - $\delta(q_0, \sqcup) = (q_{\text{accept}}, a, R)$
   - $\delta(q_1, a) = (q_{\text{reject}}, \sqcup, R)$
   - $\delta(q_1, \sqcup) = (q_0, a, L)$

   i. (2 points) Draw the machine D.

   ii. (1 point) Give the language $L(D)$ of this machine.

   iii. (1 point) Assume the tape is bound on the left side, that is there is no empty space on the tape preceding the input. Is this DTM a decider? If so, explain why. If not, give a word. that shows it is not.

2. Consider the following Turing machine T. You may assume all transitions that have not been drawn lead to a rejecting state.



   (a) (3 points) Give a word $w \in L(T)$ of length 4 that starts with m and ends with a. Give a sequence of state configurations to show your word is in the language.

   (b) (1 point) Is T deterministic or not? Explain your answer in at most 3 lines.

   (c) (2 points) Change at most 1 label of an edge so that the word *aix* is accepted. For your answer, you only need to give what edge needs to change and what the new label should be.

3. (6 points) Consider the following two languages. Both languages are recognisable. Explaining why is worth 2 points for each. Furthermore one of the languages is decidable. Picking the right one is worth 1 point and showing it is decidable is worth 1 point.

   (a) $\mathit{MIN}_{\mathit{CFG}} = \{\langle G \rangle \mid \text{G is a minimal CFG}\}$. We say a grammar is minimal if no rule can be removed without changing the language of G.

   (b) $\mathit{ALL}_{\mathit{DFA}} = \{\langle A \rangle \mid \text{A is a DFA and } L(A) = \Sigma^*\}$

4. For each of the following claims, state whether or not the claim is true and explain your answer in at most 5 lines per claim. There is no need for full proofs.
   (a) (2 points) The alternative to the word-acceptance problem A_{TM} called PUZZLE$_{\mathsf{TM}} = \{\langle M\rangle \mid$ M is a TM and M accepts the word *puzzle*$\}$ is decidable.
   (b) (2 points) The problem $W_{\mathsf{TM}} = \{\langle M\rangle \mid$ M is a TM and M is the only TM that accepts $L(M)\}$ is co-Turing-recognisable.

5. Consider three languages A, B and C.
   (a) (2 points) If A is decidable and $B \subseteq A$ and $A \subseteq C$ hold, then B and C are both decidable. Is this true? If so, prove it. If not, give a counterexample.
   (b) (2 points) If $A \leq_{\mathsf{m}} B$ and $B \leq_{\mathsf{m}} C$ and B is undecidable, what, if anything, can we conclude about A and C?

6. (5 points) Use Rice's theorem to show the following language is undecidable. NB: Stating the two requirements of Rice's theorem is worth 2 points, applying the theorem correctly is worth the other 3.

$$L = \{\langle M\rangle \mid \text{M accepts at least two different strings of two different lengths}\}$$

7. (5 points) Suppose we have the following language:

$$L = \{\langle M, w\rangle \mid M \text{ writes an } a \text{ on the tape when it is run on } w\}$$

   We intend to prove that this language is undecidable. To that end we use a direct reduction from A_{TM} to L. Assume for the sake of contradiction that there is a TM R that decides L.

   You should now construct a TM F that uses R to decide A_{TM}.

End of the exam