DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

Midterm CSE2315
Automata, Computability and Complexity

March 12, 2020, 2 hours

- Total number of pages (without this cover page): 2.
- This exam consists of 7 open questions, the weight of each subquestion is indicated on the exam.
- Consulting handouts, readers, notes, books or other sources during this exam is prohibited. The use of electronic devices such as calculators, mobile phones etc is also prohibited.
- A single exam cannot cover all topics, so do not draw conclusions based on this exam about topics that are never tested.
- Formulate your answers in correct English and write legibly (use scrap paper first). Do not give irrelevant information, this could lead to a deduction of points.
- Before handing in your answers, ensure that your name and student number is on every sheet of paper.

Question:	1	2	3	4	5	6	7	Total
Points:	6	7	5	8	7	3	3	39

1. Consider a DFA $D = (Q, \Sigma, \delta, q_0, F)$, with:

   - $Q = \{q_0, q_{182}, q_{42}, q_1\}$
   - $\Sigma = \{a, b, c\}$
   - $F = \{q_{182}\}$
   - δ is represented by the following table:

	a	b	c
q_0	q_{182}	q_{182}	q_{42}
q_1	q_1	q_0	q_{182}
q_{42}	q_0	q_1	q_1
q_{182}	q_{42}	q_{42}	q_1

   (a) (2 points) Give a transition diagram that corresponds to D.

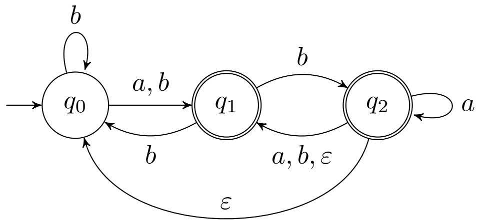
   (b) (2 points) Give a word w of length 3 that is not in D but is in $D' = (Q, \Sigma, \delta, q_0, F \cup \{q_1\})$. Explain your answer by giving the state transitions for w.

   (c) (2 points) Now imagine that we create a DFA $D'' = (Q, \Sigma \cup \Sigma_2, \delta_1, q_0, F)$, with
   $\delta_1(q, s) = \begin{cases} \delta(q, s) & \text{if } s \in \Sigma \\ \delta_2(q, s) & \text{else} \end{cases}$ for some function $\delta_2 : Q \times \Sigma_2 \to Q$
   Describe as accurately as possible the relation between $L(D)$ and $L(D'')$ in terms of $\subset, \subseteq, =$ relations. Explain your answer.

2. (a) (2 points) Assume that we define an expanded NFA model by imposing the additional requirement that $|F| = 1$, and we call this an SNFA. Are the SNFA and NFA models equally expressive? Start your answer with "Yes" or "No". If you answer yes, explain how an arbitrary SNFA can be rewritten as an NFA and vice versa. If you answer no, give an example of a language that can be accepted by an NFA, but not by an SNFA or vice versa.

   (b) (1 point) Give an NFA of at most 4 states that accepts the word mia and rejects the word $maya$. No other restrictions on the language apply. A transition diagram suffices.

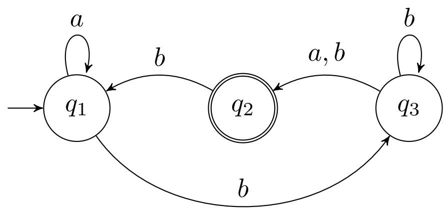
   (c) (4 points) Consider the NFA N depicted as:



   Using the method from Sipser and leaving out unreachable states, construct a new DFA D, such that $L(D) = L(N)$. A transition diagram and a short explanation suffice.

3. (a) (2 points) Provide three differences between a deterministic finite automaton (DFA) and a generalised nondeterministic automaton (GNFA).

   (b) (3 points) For the following NFA perform the first two steps in transforming it into a regular expression. To this end, perform the first step of such a transformation (transform it into a GNFA) and then for step 2 remove state q_2. Follow the algorithm provided by Sipser, and show both steps with a separate diagram.



4. (8 points) Consider the following language, where the alphabet $\Sigma = \{1, \triangle, \equiv, \square\}$:

$$L = \{(1\triangle)^n 1(\square 1)^n \equiv 1 \mid n \geq 1\}$$

This language is **not** regular. Prove this using the pumping lemma.

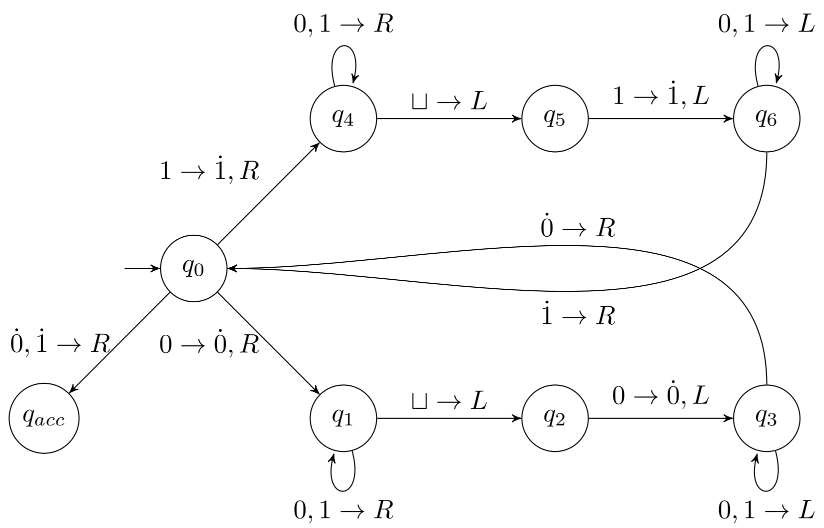
5. Consider the following rules R of a CFG $G = (V, \Sigma, R, S)$, with $V = \{S, A, B\}$:

$$S \to ASA \mid Ba$$
$$A \to bB \mid S$$
$$B \to c \mid b \mid \varepsilon$$

   (a) (1 point) Give a valid set Σ such that $|\Sigma| = 5$.

   (b) (6 points) Convert G into Chomsky Normal Form (CNF). Use the procedure from Sipser, showing intermediate steps.

6. Consider Turing machine M below with $\Sigma = \{0, 1\}$ and accepting state q_{acc}. The rejecting state and transitions to it have been omitted.



   (a) (1 point) Give a possible tape alphabet for M.

   (b) (2 points) Is there a $w \in \Sigma^*$ such that when M processes w the machine reaches configuration $q_3\dot{0}\dot{0}$? If you answer yes, give the corresponding sequence of configurations. If you answer no, explain why this configuration cannot be reached.

7. (3 points) Given alphabet $\Sigma = \{a, b\}$ and language $L = \{abw \mid w \in \Sigma^*\}$. Is it possible to construct a Turing machine with four states that decides L? If your answer is yes, give its transition diagram. If your answer is no, argue why this is not possible.

End of the exam