

Partial Exam 3 for Analysis 1 (AM1040)
Wednesday, January 26, 2022, 09h00 - 11h00

- Clearly write your first and last name and student number on your answer sheet.
- In this midterm exam you are not allowed to use a book, notes, or a graphical/programmable calculator (the use of a simple calculator is allowed).
- Give your answers in English if possible (because that language is common to all the markers). If needed, Dutch is acceptable as well.
- Unless explicitly stated otherwise, you are required to provide clear proofs for any statements you make. In particular, if you use a result from the book or reader, show that all the required assumptions hold, and clearly state which conclusion(s) you draw.
- This exam has 6 questions. The grade is 1+ (9 · #points)/25, rounded to tenths.

1. Calculate the following integrals:

   (a) $\int \ln(x)\sqrt{x}\,dx$ [3 pts]

   (b) $\int_0^1 x^3\sqrt{1-x^2}\,dx$ [3 pts]

2. Calculate the antiderivative of the function $f:\mathbb{R}\setminus\{-1\}\to\mathbb{R}$,

$$f(x)=\frac{3x^2+13x+14}{(x+1)(x^2+4x+7)}.$$

[5 pts]

3. Prove if the following integral converges or diverges: $\int_0^\infty \frac{\cos(x)}{e^{x^4}}\,dx$.

[4 pts]

4. Determine all $\alpha\in\mathbb{R}$ for which the function $f:(0,1]\to\mathbb{R}, f(x)=x^\alpha$ is uniformly continuous and justify your answer. [3 pts]

5. Prove the following result (Limit Criterion):
   Let $a\in\mathbb{R}$, and let the functions $f,g:[a,\infty)\to\mathbb{R}$ be integrable over $[a,b]$ for every $b>a$. Let $c\geq a$ such that $f(x),g(x)\geq 0$ for $x\in[c,\infty)$. If

$$\lim_{x\to\infty}\frac{f(x)}{g(x)}=L \text{ such that } 0<L<\infty,$$

   then $\int_a^\infty f(x)\,dx$ is convergent if and only if $\int_a^\infty g(x)\,dx$ is convergent. [3 pts]

6. Suppose that $f:[a,b]\to[1,2]$ is integrable. Prove that $\sqrt{f}$ is integrable on $[a,b]$ as well.
   You may use without proof that $\sup\{\sqrt{f(x)}:x\in I\}\leq\sqrt{\sup\{f(x):x\in I\}}$ and $\inf\{\sqrt{f(x)}:x\in I\}\geq\sqrt{\inf\{f(x):x\in I\}}$ for any interval $I\subseteq[a,b]$.

[4 pts]