Resit Reasoning and Logic (CSE1300)

2021–22

Please read the following information carefully!

- This exam consists of 11 open questions. The open questions are worth a total of 62 points and the points per open (sub) question are given in the (sub) question itself.
- The grade for this exam is computed as $1 + 9 \cdot \frac{\text{score}}{62}$.
- This exam corresponds to all non-starred sections of the book: *Delftse Foundations of Computation* (version 2.0).
- You have 180 minutes to complete this exam.
- Before you hand in your answers, check that every sheet of answer paper contains your name and student number.
- The use of the book, notes, calculators or other sources is **strictly prohibited**.
- Read every question carefully and, in the case of the open questions, give **all information** requested. Do not however give irrelevant information: this could lead to a deduction of points.
- You may write on this exam paper and take it home.
- No animals were harmed in the creation of this exam.
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Question:	1	2	3	4	5	6	7	8	9	10	11	Total:
Points:	5	8	3	6	3	6	5	6	7	6	7	62

Open questions

1. (a) (3 points) Give a Tarski World that satisfies the following properties (a picture suffices as an answer), or explain why no Tarski World can satisfy these properties.
   - $\forall x(\mathit{Circle}(x) \to \mathit{LeftOf}(x, d))$
   - $\exists x(\mathit{Circle}(x)) \land \exists x(\mathit{Square}(x)) \land \exists x(\mathit{Triangle}(x))$
   - $\forall x(\mathit{Triangle}(x) \leftrightarrow \exists y(\mathit{Square}(y) \land \mathit{AboveOf}(x, y)))$

   (b) (2 points) Is the following set of statements satisfiable? If so, prove it by providing a formal structure. If not, explain clearly why not.
   - $\forall x \exists y(P(y) \to R(x, y))$
   - $\exists y \forall x(P(y) \to R(x, y))$

2. (a) (5 points) It's winter time and the local squirrel population has all collected their food and gone into hibernation. Alfie the squirrel has taken his family and they are now in the process of rationing the food. Starting with a stash of 250 acorns and 19 chestnuts, the five squirrels (Alfie, his partner Archie, and the extended family Trilo, Regina, and Moe) now wonder if the splitting procedure they planned is sufficiently fair. The procedure works as follows:
   - Each of the squirrels takes 10 acorns to start with.
   - Then they take turns to grab two items from the stash at random, in the order they are listed above (starting with Alfie and ending with Moe).
     - If they take two acorns, they get to keep one acorn and put the other one back.
     - If they take two different things, they get to keep the acorn and put the chestnut back.
     - If they take two chestnuts, they get to keep both chestnuts, but only if they can pay an acorn for it (putting it back in the stash). If they cannot, they put both chestnuts back and they go again.
   - They stop when it is impossible to take two items from the stash.

   Who will be the last to take an item from the stash and what item will that be? As the invariant use the fact that the number of chestnuts is odd.

   (b) (3 points) Hibernation is a long affair and sometimes briefly interrupted. During those interruptions Alfie and Archie have taken to playing some tic-tac-toe on the wall of their home[1]. Being squirrels they are not exceptionally great at the game and the only thing you can really assume about their intelligence is that if they can complete their sequence of 3 x's or o's right now, they'll do it. Otherwise their move has no other strategy behind it. Furthermore Alfie always plays with the o's and Archie with the x's, with the starting player determined by whoever wakes up first.

   You check in on their game at the start of winter and see that they have only just started, with the board looking like this.

		O
	X	

   After several months you check in again and you see the game is almost done. Who will win? Explain how you derived the answer.

O		O
X	X	O
O	X	X

[1] Remember tic-tac-toe is the game played on a 3x3 grid where players take it in turn to place an x or o on the board. The first to connect three in a row, column or diagonal is the winner of the game.

3. Consider the following constants, predicates, and functions:

   - The domain of discourse is 'all objects' (including living beings).
   - $Book(x)$ means x is a book.
   - $Author(x)$ means x is an author.
   - $Write(x, y)$ means x wrote y.
   - $Read(x, y)$ means x read y.
   - g represents "Mark Gimenez".
   - c represents "The color of law".
   - f represents "Finch".

   Translate the following phrases to unambiguous natural language (English).

   (a) (1 point) $\exists x(Book(x) \land Wrote(g, x)) \to Author(g)$

   (b) (2 points) $\exists A(\forall x \in A(Wrote(g, x)) \land c \in A \land \forall x \in A(Read(f, x))$

4. (a) (2 points) Can a function be an equivalence relation? If yes, provide such a function and explain your answer. Otherwise explain why not.

   (b) (2 points) Give two functions that are not well-defined for different reasons. Explain clearly why they are not well-defined.

   (c) (2 points) Give an example of a function that is surjective but not injective and an example of a function that is injective but not surjective. Briefly explain your answers (no formal proofs required).

5. (a) (1 point) Given the sets $A = \{\emptyset\}, B = \{(1, 2), \{3\}\}, C = \{1\}$. Compute $A \times B \times C$.

   (b) (2 points) Draw a Venn Diagram for the set: $(D \cup E) \cap (F \Delta D)^c$

6. (6 points) Consider the following argument written in propositional logic:
$$\begin{array}{l} p \to r \\ q \leftrightarrow \neg(p \lor r) \\ \hline \therefore \neg q \land \neg r \end{array}$$
Construct a *full* truth table for the relevant propositions in the argument and use this truth table to show the argument is (in)valid. Explain how your truth table shows this by referring to specific rows of the truth table.

7. For each of the following claims start your answer with either *True* or *False*. Then either explain why it is true, or give a counterexample with an explanation explaining why it is false.

   (a) (2 points) Any argument in which a premise is necessary for the conclusion is logically valid.

   (b) (1 point) Any argument can be made valid by just adding a premise.

   (c) (2 points) The following English sentence is unambiguous (meaning there no two unequivalent translations to predicate logic): Brown marmots and squirrels are cute.

8. (a) (1 point) Consider that you want to prove the following claim using a proof by contrapositive. What would be the starting assumption of the proof?
   All integers divisible by 18 are not prime.

   (b) (5 points) Prove the following claim:
   For all real numbers x: if $0 < x < 1$ then $\frac{1}{x(1-x)} \geq 4$

9. Consider the following recursively defined set of words A. Letters used in the word are from the set $\{e, i, l, n\}$, the other letters used here are variables.

   I. $e \in A$

   II. $nxl \in A \land lxn \in A$, if $x \in A$

   III. $xenyi \in A$, if $x, y \in A$

   IV. Nothing else is in A.

   (a) (1 point) Give a word of length 5 (meaning five letters) starting with n that is in A, explain which rules you used to construct the word.

   (b) (6 points) Prove the following claim: $\forall w \in A(2 \nmid f(w))$ where f is a function $f : A \to \mathbb{N}$ that returns the length of the input word.

10. (a) (3 points) Give a tree that has the following properties (a drawing suffices).
    - The tree has a height of 3.
    - There are 2 nodes with 3 children.
    - The root's value is equal to the number of leafs in the tree.
    - The value of every node is odd iff the number of descendants of the node is odd.

    (b) (3 points) Give a graph $G = (V, E)$ that has the following properties (a drawing suffices):
    - $|E| = 8, |V| = 6$
    - $\{(a, b), (b, c), (k, m)\} \subseteq E$
    - The graph does *not* have a topological ordering.
    - There is a node with degree 3.

11. (a) (2 points) Give a recursive definition of a *sequence* starting with the numbers 1 and 18 where the following number is the absolute difference between the previous two plus the index in the sequence.

    (b) (5 points) Prove the following claim using mathematical induction:
    For all integers $n \geq 1$: $\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$

End of the exam