Resit Midterm Reasoning and Logic (CSE1300)

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Please read the following information carefully!

- This exam consists of 7 open questions. The open questions are worth a total of 60 points and the points per open (sub) question are given in the (sub) question itself.
- The grade for this exam is computed as $1 + 9 \cdot \frac{\text{score}}{60}$. Note that you require a grade of at least 5 out of 10 to pass the course (assuming your average course grade is at least 5.75).
- This exam corresponds to chapters 1 to 3 of the book *Delftse Foundations of Computation* (version 1.01), with the exception of the topic of induction.
- You have 90 minutes to complete this exam.
- Before you hand in your answers, check that every sheet of paper contains your name and student number, as well as the total number of sheets of paper that you hand in.
- The use of the book, notes, calculators or other sources is **strictly prohibited**.
- Read every question properly and in the case of the open questions, give **all information** requested. Do not however give irrelevant information, this could lead to a deduction of points.
- You may write on this exam paper and take it home.
- Exam is ©2018 TU Delft.

Question:	1	2	3	4	5	6	7	Total:
Points:	6	5	10	9	7	15	8	60

1. For this question you need to translate claims from natural language to predicate logic. Make sure to define all predicates you introduce and to include all of the information from the original statement in your translation.
   (a) (3 points) The BEP[1] is not part of the 2018 curriculum, but another project is.
   (b) (3 points) All magicians wear a silk hat or a rose.

2. Translate the following two claims to natural language (English). The predicates and domain used in our claims are as follows:

   - The domain of discourse is "all objects" (including living beings).
   - $Genius(x)$ means: x is a genius.
   - $TimeMachine(x)$ means: x is a time machine.
   - $Owns(x, y)$ means: x owns y.
   - d is "the Doctor" (this represents a single specific person).

   (a) (2 points) $\forall x(Genius(x) \land \exists y(TimeMachine(y) \land Owns(x, y)))$
   (b) (3 points) $\forall x(Genius(x) \rightarrow (x = d)) \land \neg\exists y(TimeMachine(y) \land \neg\exists x(\textbf{Genius}(x) \land Owns(x, y)))$

3. (a) (7 points) Consider the new operator $\circ$ with the truth table:

p	q	$(p \circ q)$
0	0	0
0	1	1
1	0	0
1	1	1

   Are $(p \rightarrow q) \circ (\neg r \leftrightarrow p)$ and $p \circ (\neg p \leftrightarrow r)$ equivalent? Explain your answer using a truth table.

   (b) (3 points) Rewrite $(p \land q) \rightarrow \neg(r \lor q)$ to DNF. Simplify your result as much as possible.

4. (a) (4 points) Consider the following description of a Tarski World. There is one 'colour' in this world, which we call *filled* (use whatever your colour you are writing the exam in). Does an instance of a Tarski World exist with these properties? If so, give one with a domain of at most 5 elements. Make sure to label the objects with their name and that for filled objects these names are readable. If no such instance exists, explain why not in at most 5 lines.

   - $\forall x(Circle(x) \rightarrow \neg Filled(x))$
   - $\exists x(Circle(x)) \land \exists x(Filled(x))$
   - $RightOf(a, b)$
   - $LeftOf(a, b) \lor Square(c)$

   (b) (5 points) Consider the domain $\{a, b\}$ and the statements:
   $\forall x(P(x) \lor Q(x))$
   $\exists x \exists y(R(x, y))$
   $\forall x \forall y(R(x, y) \rightarrow (Q(x) \leftrightarrow P(x)))$

   Show that it is possible to create a structure A in which all of the above statements hold, but $P^A \neq Q^A$. Explain why each of the claims hold. Answer in at most 10 lines.

[1]Bachelor Final Project

5. (a) (1 point) Give an example of a number that is in $\mathbb{R}$, but not in $\mathbb{Q}$.

   (b) (4 points) Consider the following description of the predicate P which should describe if a number is prime.
   $\forall x \in \mathbb{N}(P(x) \leftrightarrow \forall n \in \mathbb{N}(n > 4 \rightarrow n \nmid x))$
   This description is incorrect. Give one example of a number that has the property P even though it is not prime, and give one example of a number that does not have the property P even though it is prime. Explain both of your answers in at most 5 lines each.

   (c) (2 points) Consider the following claim: For all fractions $q = \frac{a}{b}$, if a is even or if b is even, then $P(q)$ and $R(q)$ hold. Consider now the following proof:

   Proof.

   1. Take arbitrary $q \in \mathbb{Q}$, such that $q = \frac{a}{b}$ for some $a, b \in \mathbb{Z}$ where $b \neq 0$ and a or b is even.
   2. We now use a proof by division into cases.
   3. Consider a case where a is even.
   4. <Derive that $P(q)$ holds.>
   5. Consider a case where b is even.
   6. <Derive that $R(q)$ holds.>
   7. Consider a case where both a and b are even.
   8. <Derive that $P(q)$ and $R(q)$ hold.>
   9. Since q was arbitrary it holds for all $q = \frac{a}{b} \in \mathbb{Q}$ that if a or b is even, then $P(q)$ and $R(q)$ hold.

   QED

   This proof is not valid for the given claim. Explain why it is not valid and what we should change for the proof to become valid. Include references to the line numbers of the proof that should be changed. Answer in at most 8 lines.

6. (a) (8 points) Given a right angled triangle with sides a and b and hypotenuse[2] c, we know that Pythagoras' Theorem tells us that $a^2+b^2 = c^2$. Use a proof by contradiction to prove that for all right-angled triangles $a + b > c$.

   (b) (5 points) Prove the following claim for all integers n: if n is a prime larger than 2, then $n + 1$ is not prime.

   (c) (2 points) Give a counterexample to the following claim about all integers n: $3 \nmid n \rightarrow 4 \mid (n^3 + 4n^2 + 3n)$. Explain how your example disproves the claim in at most 5 lines.

7. (a) (2 points) Consider the following recursively defined formula:

$$f(n) = \begin{cases} \pi & \text{if } n \leq 0 \\ f(n-3) + \pi & \text{if } n \text{ is odd} \\ f(n/2) - \pi & \text{if } n \text{ is even} \end{cases}$$

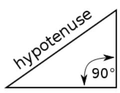
   Compute $f(0), f(2), f(9)$.

   (b) Give a recursively defined sequence or formula for each of the following descriptions:

   i. (3 points) All negative integers that end in a 1.

   ii. (3 points) The first item of the sequence is the integer 2, the second item is the first plus one. Every next item in the sequence is the sum of the previous two entries, plus the first item in the sequence.

[2]Remember that the hypotenuse is the side opposite the right angle. Visually (image from Wikipedia):



End of the exam