# Midterm Reasoning and Logic (CSE1300)

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**Please read the following information carefully!**

- This exam consists of 8 open questions. The open questions are worth a total of 68 points and the points per open (sub) question are given in the (sub) question itself.
- The grade for this exam is computed as $1 + 9 \cdot \frac{\text{score}}{68}$. Note that you require a grade of at least 5 out of 10 to pass the course (assuming your average course grade is at least 5.75).
- This exam corresponds to chapters 1 to 3 of the book *Delftse Foundations of Computation* (version 1.1), with the exception of the topic of structural induction.
- You have 120 minutes to complete this exam.
- Before you hand in your answers, check that every sheet of paper contains your name and student number.
- The use of the book, notes, calculators or other sources is **strictly prohibited**.
- Read every question properly and in the case of the open questions, give **all information** requested. Do not however give irrelevant information, this could lead to a deduction of points.
- You may write on this exam paper and take it home.
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| Question: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | Total: |
|---|---|---|---|---|---|---|---|---|---|
| Points: | 12 | 7 | 8 | 6 | 7 | 6 | 17 | 5 | 68 |

1. (a) (2 points) Create a truth table for: $\neg(p \leftrightarrow q) \vee (\neg p \wedge q)$.
   (b) Consider a new type of operator that operates on three propositions at a time. This ternary operator is defined by the following truth table:

| p | q | r | $p \xrightarrow{q} r$ |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

   Someone argues that $(q \xrightarrow{p} q) \vee r \equiv \neg(p \vee r) \xrightarrow{q} r$ holds.
      i. (6 points) Create the full truth table for both propositions.
      ii. (1 point) Describe how we can derive the (in)validity of this equivalence from your truth table.
   (c) (3 points) Rewrite $(p \vee q) \rightarrow \neg(r \wedge q)$ to DNF. Simplify your result as much as possible.

2. For each of the following claims, either explain why they are true, or give a counterexample. Start your answer with either the word "True" or "False" indicating which of the two options applies.
   (a) (2 points) It is *not* possible for some proposition $p$ to be both sufficient and necessary for some proposition $q$.
   (b) (2 points) Every argument that can be represented in propositional logic, can be represented in predicate logic.
   (c) (3 points) You can prove a property $P(x)$ holds for all even integers $\geq 3$ using induction. If you answer true, explain what the base case and induction step should look like. If you answer false, give a counterexample and a brief explanation.

3. For this question you need to translate claims from natural language to predicate logic. Make sure to define all predicates you introduce and to include all of the information from the original statement in your translation.
   (a) (2 points) Stefan loves CSE1300.
   (b) (3 points) There is a TA that has a blue badge.
   (c) (3 points) When you grade an exam, you eat a pepernoot.[1]

4. Translate the following two claims to natural language (English). The predicates and domain used in our claims are as follows:
   - The domain of discourse is "all objects" (including living beings).
   - $Song(x)$ means: $x$ is a song.
   - $Artist(x)$ means: $x$ is an artist.
   - $Better(x, y)$ means: $x$ is better than $y$.
   - $Performs(x, y)$ means: $x$ performs $y$.
   - $t$ is Tom Lehrer.
   - $e$ is The Element Song.

   (a) (1 point) $Song(e) \wedge \neg Artist(e)$
   (b) (1 point) $\neg\exists x(Artist(x) \wedge Better(x, t))$.
   (c) (4 points)
   $\forall x((Song(x) \rightarrow \exists y(Artist(y) \wedge Performs(y, x))) \wedge (Artist(x) \rightarrow \exists y(Song(y) \wedge Performs(x, y))))$

5. (a) (4 points) Consider the following set of predicates over the domain $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
   - $\forall x(P(x) \leftrightarrow (x \mid 2))$
   - $\forall x(Q(x) \leftrightarrow \exists z(z < 4 \wedge (3 \mid (x + z))))$
   - $\forall x \forall y(R(x, y) \leftrightarrow ((x = y + 2) \wedge \exists z((z > 4) \wedge (x \mid (z + y)))))$

   Give the truth sets for $P$, $Q$, and $R$.

[1] "A pepernoot" is a small "cookie" Dutch people commonly eat from September to December.

(b) (3 points) Draw a Tarski World with at least 4 objects and at most 6 objects, which satisfies the following criteria. Note that we have only two "colours": Filled (use whatever colour you are doing the exam with) and Empty (do not colour it). Briefly explain how your drawing satisfies the criteria.

- $\exists x(\mathit{Square}(x) \wedge \exists y(\mathit{LeftOf}(x, y)))$
- $\mathit{Filled}(f)$
- $\forall z(\mathit{Filled}(z) \rightarrow (\exists x(\mathit{LeftOf}(x, z) \wedge \mathit{Triangle}(x)) \vee \exists y(\mathit{RightOf}(y, z) \wedge \mathit{Circle}(z))))$

6. (a) (3 points) Is the following argument valid? If so, explain why in at most 5 lines. If not, give a formal structure to prove it is not valid and briefly explain how your structure shows this.

$\forall x(P(x) \vee Q(x))$
$\forall x(P(x) \rightarrow \exists y(R(x, y)))$
$R(a, b)$
$\therefore P(a)$

(b) (3 points) Is the following set of predicates satisfiable? If so, give a formal structure to prove it and briefly explain how your structure shows this. If not, explain why not in at most 5 lines. (Note that this is not the same set as in the previous question!)

$\forall x(P(x) \wedge Q(x))$
$\forall x(P(x) \rightarrow \forall y(R(x, y)))$
$\neg R(a, b)$

7. (a) (7 points) Prove the following claim for all integers $n$: if $n^2 + 6n - 3$ is even, then $n$ is odd.

(b) (7 points) Prove the following claim for all positive integers $n$: if $3 \nmid n$ then $3 \mid n^2 + 2$.

(c) (3 points) Consider the following faulty proof by contradiction for the claim: if $n$ is prime, then $3n$ is prime.

*Proof.* We introduce $P(n)$ as the predicate for $n$ is prime.

1. For the sake of contradiction, assume there is an integer $k$ such that $\neg P(k) \wedge P(3k)$.
2. Since $\neg P(k)$ we know that there is some number $c$ such that $c \mid k$.
3. Thus $k = c \cdot a$ for some integer $a$.
4. And $3k = 3c \cdot a = d \cdot a$, thus $d \mid 3k$.
5. Thus $\neg P(3k)$.
6. This forms a contradiction with our assumption in step 1, thus there cannot be such an integer $k$.
7. Thus $P(n) \rightarrow P(3n)$ holds for all $n$.

QED

Which of the steps contain(s) a mistake? Clearly describe what the mistake(s) is (are).

8. (a) (2 points) Consider the following recursively defined formula:

$$f(n) = \begin{cases} 2^n & \text{if } n \leq 0 \\ 2 * f(n-1) & \text{if } n \text{ is odd} \\ f(n-2) + f(n/2) & \text{if } n \text{ is even} \end{cases}$$

Compute $f(-1), f(4), f(11)$.

(b) (3 points) Consider the following two formula:

$$f(n) = \sum_{i=1}^{n} (i + n) \qquad g(n) = \sum_{i=1}^{n} f(i)$$

Compute $f(4)$ and $g(3)$.

End of the exam