Midterm Exam(ple)

Computer Organisation

Please read the following information carefully!

- On a real exam, this is gonna be full of info you should read.
- It should take you 90 minutes to complete this practice exam.
- The answers provided are meant for checking your results, not as example answers. We expect you to show your work and provide full explanations and calculations for your answers.

1. (300 points) Which of the following equations **are** equal to each other?

   X $ac + \bar{a}\bar{b}\bar{c} + \bar{b}\bar{c}\bar{d}$

   X $c\bar{d}a + cda + \bar{a}\bar{b}\bar{c} + \bar{b}\bar{c}\bar{d}$

   ☐ $acd + \bar{a}dc + \bar{a}\bar{b}\bar{c} + \bar{b}\bar{c}\bar{d}$

   ☐ $ac + \bar{a}\bar{b} + \bar{c}\bar{d} + \bar{b}\bar{c}\bar{d}$

2. (300 points) You're applying for a software developer position at the tech giant company **Ducks & co.** and you got invited to an interview. In the invitation you were asked to prepare an answer to the following problem:

   > A design for a digital circuit is needed which adds three input bits: a, b, and c. The output of this circuit should be the sum bit S. What formula can be used to create this circuit? Show how you got to the formula.

Solution:

a	**b**	**c**	**S**
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

The following formula can be derived: $S = \bar{a}\bar{b}c + \bar{a}b\bar{c} + a\bar{b}\bar{c} + abc$

3. (400 points) While working on the Self-study assignments, your teammate accidentally spills coffee over your Karnaugh map. Unfortunately, it has become completely unreadable and you will have to redo the whole map to determine the minimal sum-of-products.

   The formula that you were trying to minimize is $C\bar{D} + A\bar{B} + \bar{A}\bar{B}\bar{D} + ABC$.

   (a) Fill out the Karnaugh-map below.

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	0	0	1
11	0	0	1	1
10	1	1	1	1

Solution:

(b) Determine the minimal sum-of-products.

> **Solution:** $y = \bar{B}\bar{D} + C\bar{D} + AC + A\bar{B}$

(c) Explain in maximum two sentences how you can get a minimal sum-of-products formula from a Karnaugh map.

> **Solution:** Make the biggest groups / circles in the karnaugh map possible (1x1, 1x2, 2x2, 1x4, 2x4, 4x4), until all 1s are covered. These are the parts of the formula.

4. (300 points) We are going to implement CO-hash. The idea is that we are going to change your student number such that you do not have to remember 7 digits anymore, but something "simpler" to remember that will (most likely) still be unique for at least your cohort! So what you need to do is the following:

   1. Take your student number.
   2. Take the last 2 digits modulo 36; this is your *base*.
      If that results in `00`, `01`, `02`, `08`, `10`, or `16` add `13` (so change to `13`, `14`, `15`, `21`, `23`, or `29` respectively).
   3. Convert your student number to this new base.

   Make sure to write the whole calculation.

5. (400 points) Mr. Hugtenburg wants to track the coconut trees that Coatis visit over the course of a week. He decides he wants to uniquely identify each Coati-Tree combination, with the weekday and hour that the Coati visited the tree.

   Details:

   - Mr. Hugtenburg estimates there are about 4 million Coatis.
   - There are about 8 million coconut trees in reach of the Coatis.
   - Mr. Hugtenburg has found that Coati visit at most 75 different trees a day.
   - (As you know) there are 24 hours in a day and there are 7 days in a week.
   - The numbers must be represented with max 15 characters.

   You are asked for advice on the data representation of these numbers. Explain your answers.

   (a) What is the minimum number of bits required to represent these numbers?

   (a) **53 bits**

   > **Solution:** 4 million $\approx 2^{22}$ coati
   > 8 million $\approx 2^{23}$ trees
   > $24 \approx 2^5$ hours
   > $7 \approx 2^3$ days
   > (Or $24 \cdot 7 = 168$ hours $\rightarrow 168 \approx 2^8$ hours)
   > Minimum: $22 + 23 + 5 + 3 = 53$ bits

   (b) Which number representation should be used?

   (b) **hexadecimal**

   > **Solution:** $53/15 \approx 3.5$ and $2^{3.5} \approx 11$ so we need at least radix-12 (e.g. hexadecimal would work). If $53/15$ is rounded to 4, we get $2^4 = 16$, so again, hexadecimal would work.

(c) What is the maximum amount of data he can expect after tracking them for a day? Round to whole GiB.

(c) **2 GiB**

Solution: Max data Stefan can expect is:
53 bits $\cdot$75 trees/day $*2^{22}$ coatis
$= 3975 \cdot 2^{22}$
$\approx 2^{12} * 2^{22}$
$= 2^{34}$ bits
$= 2^{31}$ Bytes
$= 2$ GiB

6. (300 points) Romeo Duck and Juliet Shark got into a bit of a fight. Romeo Duck says that if you use BCD you cannot just add these numbers together as the result will not be BCD anymore. Juliet Shark on the other hand says that it can be done, but you just need to add 6 to the resulting BCD digit if there is overflow (e.g.: if the addition of 2 BCD digits results in `1111` (15) then add `0110` (6), which results in `0001 0101`, or 15).

   Given the numbers 69 and 42, your task is to preserve love and peace in the animal kingdom and prove them both right.

   (a) Prove Romeo Duck right by showing that regular binary addition of these two numbers does not result in a valid BCD number.

   Solution:

```
    1
  0110 1001
+ 0100 0010
-----------
  1010 1011
```

   → Not a valid BCD because (10)(11)

   (b) Prove Juliet Shark right by showing that her solution works to get the correct result in BCD.

   Solution:

```
   11 1 11
  1010 1011
+ 0110 0110
-----------
0001 0001 0001
```

   → Valid and correct BCD because 69 + 42 = 111

7. (300 points) Ms. 'Sweet sixteen' has designed her own floating point standard, which starts off with a sign bit, followed by a 5-bit exponent (in excess-32), and a 10 bit mantissa with a 'hidden' bit just like big brother IEEE 754.

   Consider the following number: `0 11001 1101010000`

   Derive the resulting number (bit string) when multiplying it with the scalar *5*.

   Hint: multiplication can be expressed in terms of shifts and additions.

   Show your work!

   Solution: $5 * fp = 4 * fp + 1 * fp$

```
1 fp =    1.1101010000
4 fp = 111.01010000
_____________________
       1001.0010010000
```

   Now we need to normalize: shift right until we have 1. and increment the exponent to arrive at `0 11100 0010010010`.

8. (400 points) Ducks decided some 5 years ago that they need to start a census to determine once and for all whether there are more sharks or more ducks in the world. They discovered that in those 5 years the number of ducks doubled every year. They store the amount of ducks in CO-42 which has the following properties:

   - 2 sign bits
   - 8 exponent bits, excess-127 (same rules and special numbers as IEEE-754)
   - 32 mantissa bits

   There were 65536 ducks when they started counting. In what year will the amount of ducks no longer fit?

   8. **2126**

   Solution: 65536 ducks = 2^{16}
   biggest exponent = 127
   $127 - 16 = 111$ years; started 5 years ago, so the year 2126

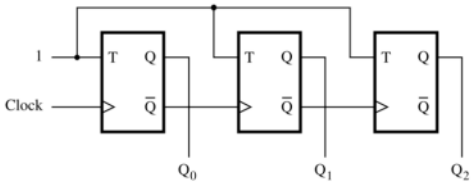
9. (200 points) Select all that apply:

   ☐ A clock can be both edge and level triggered.

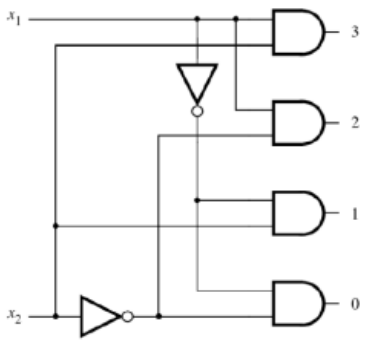
   X A clock can be both rising and falling edge triggered.

   ☐ The difference between D latch and D flip-flop is that D flip-flop does not need a clock.

   ☐ Level triggered is preferred over edge triggered.



(a)



(b)

Figure 1

10. (200 points) Consider the diagrams in Figure 1. What do they represent?

A. (a) Counter and (b) Multiplexer

B. (a) Decoder and (b) Counter

C. (a) Counter and (b) Decoder

D. Both are counters

10. _____

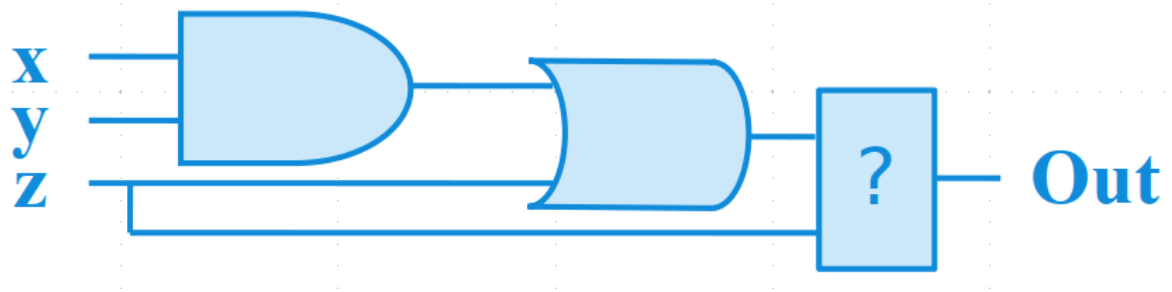


Figure 2

11. (200 points) Which gate needs to be inserted at the ?-box in Figure 2 in order to get the desired output as listed in the following truth table?

X	Y	Z	Out
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

A. AND

B. NAND

C. OR

D. NOR

11. _____

12. (300 points) We are designing a new ISA. This new laptop ISA will be specifically targeting the needs of the students. This means the ISA needs to be efficient at streaming video and copying bytes. What has been decided so far is the following:

    - minimum 4.2; maximum 42 GiB of RAM
    - 42 registers of 64 bits
    - 42 instructions should be enough
    - It should support operations with 2 memory operands

    The length of the instructions has not been decided yet, but to make things easy it will be fixed length instructions. How long would you make the instructions and why?

    12. **72 or 80**

    Solution: 42 GiB $\rightarrow 2^{36}$ for byte addressable 42 regs $\rightarrow$6 bits 42 instructions $\rightarrow$6 bits

    It needs to be efficient at copying bytes, so we need memory to memory instructions at least. Registers are 64 bits, we probably do not need byte addressable, so 33 bits should work (word-addressable, where a word is 64-bit) for a memory address.

    $6 + 33 + 33 = 72$ bits

    or

    $6 + 36 + 36 = 78$ bits

    We want a multiple of 8 bits for our instruction length, so either 72 or 80 bit instructions.

13. (400 points) You are given the following piece of 'Assembly x86-64' code:

```
1          popq    %rdi
2          popq    %rsi
3          call    foo
4          incq    %rax
5          pushq   %rax
6
7          movq    $0, %rdi
8          call    exit
9
10         foo:
11             pushq   %rbp
12             movq    %rsp, %rbp
13
14             addq    %rsi, %rdi
15             pushq   %rdi
16             pushq   %rdi
17             movq    %rsi, %rax
18
19             movq    %rbp, %rsp
20             popq    %rbp
21             ret
22
```

    Before `line 1` is executed, the 8-byte aligned stack looks like this:

Address	Contents	
464	"COCONUT="	RBP
456	"=NOTANUT"	
448	20	
440	22	RSP
432	0	
424	837	
416	351	
408	656	

(a) What will the stack look like after `line 17` is executed? Fill in the following table:

Solution:

Address	Contents	
464	"COCONUT="	
456	"=NOTANUT"	
448	`<ret>`	
440	`<RBP>`	`RBP`
432	42	
424	42	`RSP`
416	351	
408	656	

(b) What will the stack look like after `line 5` is executed? Fill in the following table:

Solution:

Address	Contents	
464	"COCONUT="	`RBP`
456	"=NOTANUT"	
448	21	`RSP`
440	`<RBP>`	
432	42	
424	42	
416	351	
408	656	

End of exam.