Computer Graphics - CSE2215 - 2021/2022

Transformations and Homogeneous Coordinates

All questions assume the course's definition for points at infinity

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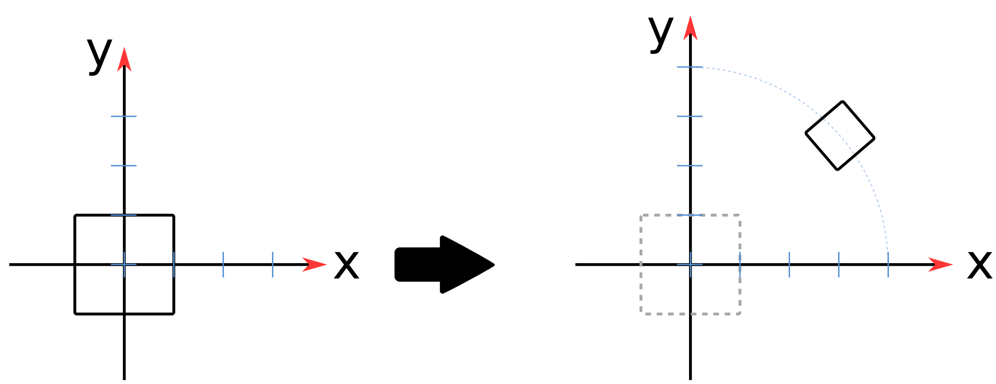
1. (1 point) We wish to transform the square in the left image into the square of the right image using 3x3 transformation matrices. The original square has sides of 2 unit length and is centred at the origin. The transformed square has half the dimensions (1 unit length) and is centred at position $(2\sqrt{2}, 2\sqrt{2})$. By which of the following sequences of transformations can we multiply the vertices of the square on the left image to obtain the one on the right?



i. $\begin{bmatrix} cos(45^o) & -sin(45^o) & 0 \\ sin(45^o) & cos(45^o) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2\sqrt{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

ii. $\begin{bmatrix} cos(45^o) & -sin(45^o) & 0 \\ sin(45^o) & cos(45^o) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iii. $\begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(45^o) & -sin(45^o) & 0 \\ sin(45^o) & cos(45^o) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

iv. $\begin{bmatrix} cos(45^o) & -sin(45^o) & 0 \\ sin(45^o) & cos(45^o) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 8 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

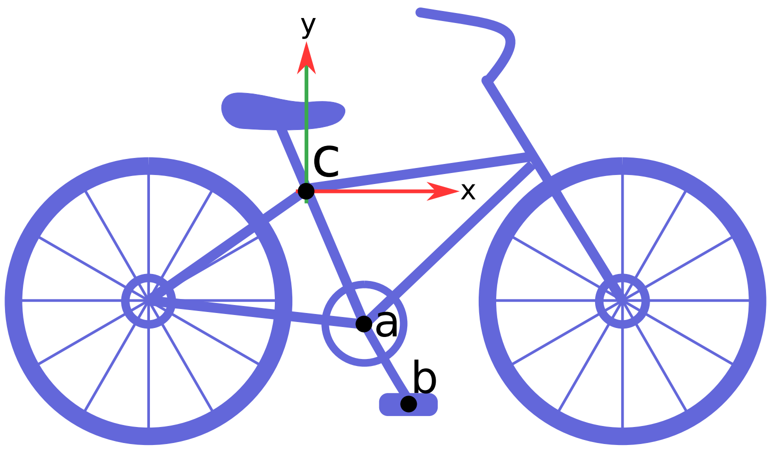
v. $\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(45^o) & -sin(45^o) & 0 \\ sin(45^o) & cos(45^o) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

A. Skip Question

B. only (i) C. only (ii) D. only (iii) E. only (iv)

F. only (v) G. only (i) and (iii) H. only (ii) and (iv)

2. (1 point) In this question you are asked to simulate a 2D bicycle, as illustrated below. The centre of the bicycle c is initially at position $(0, 0)$ and the centre of the crank is at position $a = (a_x, a_y)$, hence, the centre of the pedal b rotates around point a. For every full turn of the pedal the bike moves 1 unit forward (positive x-axis). **Given a rotation of the pedal by angle θ, which of the following transformations should we apply to point b to achieve such movement?** You can assume θ is a negative value so the pedal rotates clockwise.



A. Skip Question

B. $\begin{bmatrix} 1 & 0 & \frac{|\theta|}{2\pi} + a_x \\ 0 & 1 & a_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -a_x \\ 0 & 1 & -a_y \\ 0 & 0 & 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 & 0 & \frac{|\theta|}{2\pi} - a_x \\ 0 & 1 & -a_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & a_x \\ 0 & 1 & a_y \\ 0 & 0 & 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 & 0 & a_x + \frac{|\theta|}{2\pi} \\ 0 & 1 & a_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix}$

E. $\begin{bmatrix} 1 & 0 & a_x + \frac{|\theta|}{2\pi} \\ 0 & 1 & a_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) & b_x \\ sin(\theta) & cos(\theta) & b_y \\ 0 & 0 & 1 \end{bmatrix}$

F. $\begin{bmatrix} 1 & 0 & a_x \\ 0 & 1 & a_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{|\theta|}{2\pi} - a_x \\ 0 & 1 & -a_y \\ 0 & 0 & 1 \end{bmatrix}$

G. $\begin{bmatrix} 1 & 0 & a_x + b_x \\ 0 & 1 & a_y + b_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & \frac{|\theta|}{2\pi} - a_x \\ 0 & 1 & -a_y \\ 0 & 0 & 1 \end{bmatrix}$

H. $\begin{bmatrix} 1 & 0 & \frac{|\theta|}{2\pi} + a_x \\ 0 & 1 & a_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta) & -sin(\theta) & 0 \\ sin(\theta) & cos(\theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -(a_x + b_x) \\ 0 & 1 & -(a_y + b_y) \\ 0 & 0 & 1 \end{bmatrix}$

Cameras and Projections

All questions assume the course's definition for points at infinity

3. (1 point) We want to create a virtual portrait pinhole camera. In order to do so, we place a virtual camera at the origin facing the $-z$ axis and people are asked to keep their heads inside of a cube with side length 4 units, centred at $(0, 0, -10)$. What is the maximal focal length (f) in order to entirely capture the cube containing a head? Consider the aspect ratio of the camera to be one.

   A. Skip Question B. 1 C. 2 D. 3 E. 4 F. 5 G. 6 H. 8

OpenGL

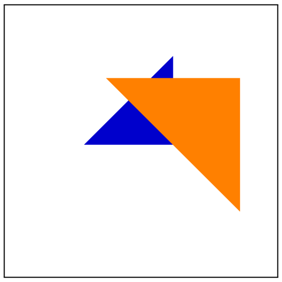
4. (1 point) If we modify the projection matrix on the left to the one on the right and run the following OpenGL code below, which of the following images is the corresponding result? You can assume the following parameters for the projection matrix: aspect = 1; $f = 1$; near = 0.01, far = 100.

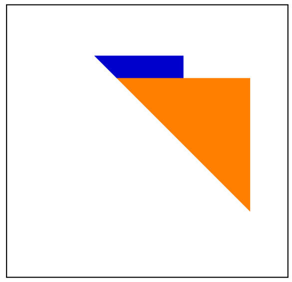
$$\begin{bmatrix} \frac{f}{\text{aspect}} & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & -\frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2.0f*\text{near}*\text{far}}{\text{far}-\text{near}} \\ 0 & 0 & -1.0 & 0 \end{bmatrix} \longrightarrow \begin{bmatrix} -\frac{f}{\text{aspect}} & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & -\frac{\text{far}+\text{near}}{\text{far}-\text{near}} & -\frac{2.0f*\text{near}*\text{far}}{\text{far}-\text{near}} \\ 0 & 0 & -1.0 & 0 \end{bmatrix}$$

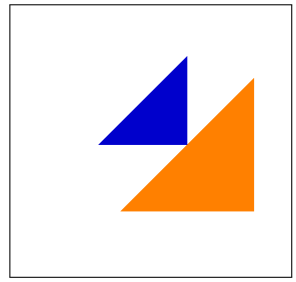
Please note that in the code below OpenGL reads matrices in column order (transposed)

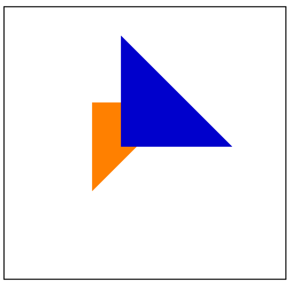
```
glm::mat4 mm    (    1,     0,      0,  0,
                     0,     1,      0,  0,
                     0,     0,      1,  0,
                  -0.2,   0.5,   -1.5,  1 );

glLoadMatrixf(glm::value_ptr(mm));

glBegin(GL_TRIANGLES);
glColor3f(0.0f, 0.0f, 0.8f);
glVertex3f (-0.5f, -0.5f, 0.0f);
glVertex3f ( 0.5f, -0.5f, 0.0f);
glVertex3f ( 0.5f,  0.5f, 0.0f);
glEnd();

mm = glm::mat4(   1,   0,   0,   0,
                  0,  -1,   0,   0,
                  0,   0,   1,   0,
                0.2,   0,  -1,   1 );

glLoadMatrixf(glm::value_ptr(mm));

glBegin(GL_TRIANGLES);
glColor3f(1.0f, 0.5f, 0.0f);
glVertex3f (-0.5f, -0.5f, 0.0f);
glVertex3f ( 0.5f, -0.5f, 0.0f);
glVertex3f ( 0.5f,  0.5f, 0.0f);
glEnd();
```

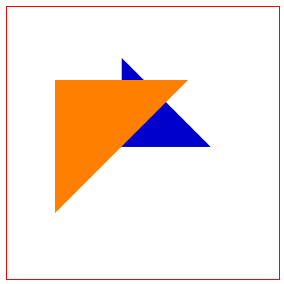
A. Skip Question

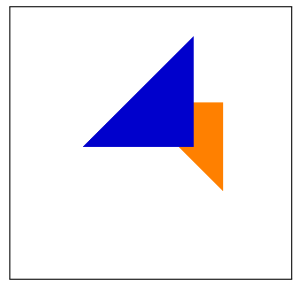
B. 

C. 

D. 

E. 

F. 

G. 

Rasterization and Interpolation

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5. (1 point) The vertices of a triangle T in homogeneous coordinates are as following: $v_0 = (0, 0, 1)$, $v_1 = (1, 0, 1)$ and $v_2 = (0, 1, 1)$. Point p in triangle T has barycentric weights $\alpha = \beta = \gamma = \frac{1}{3}$ for vertices v_0, v_1 and v_2, respectively. If we transform the vertices of T using the matrix below but keep p in the same position, what are the new barycentric coordinates of p?

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A. Skip Question

B. $\alpha = \frac{1}{3}; \beta = \frac{1}{3}; \gamma = \frac{1}{3}$ C. $\alpha = \frac{1}{4}; \beta = \frac{1}{4}; \gamma = \frac{1}{2}$ D. $\alpha = \frac{1}{2}; \beta = \frac{1}{4}; \gamma = \frac{1}{4}$

E. $\alpha = \frac{1}{2}; \beta = \frac{1}{3}; \gamma = \frac{1}{6}$ F. $\alpha = \frac{1}{3}; \beta = \frac{1}{6}; \gamma = \frac{1}{2}$ G. $\alpha = \frac{3}{5}; \beta = \frac{2}{5}; \gamma = \frac{1}{5}$

H. $\alpha = \frac{1}{5}; \beta = \frac{2}{5}; \gamma = \frac{3}{5}$

6. (½ point) Given a triangle P_1, P_2, and P_3 and function $f(\beta, \gamma) := P_1 + \beta(P_2 - P_1) + \gamma(P_3 - P_1)$, which covers all points in the plane containing the triangle. Which of the following statements is correct?

   i. If two points $A := f(\beta_0, \gamma_0)$ and $B := f(\beta_1, \gamma_1)$ are inside the triangle, then $C := f((\beta_0 + \beta_1)/2, (\gamma_0 + \gamma_1)/2)$ is inside the triangle.
   ii. If two points $A := f(\beta_0, \gamma_0)$ and $B := f(\beta_1, \gamma_1)$ are outside the triangle, then $C := f((\beta_0 + \beta_1)/2, (\gamma_0 + \gamma_1)/2)$ can be inside the triangle.
   iii. A point $A := f(\beta_0, \gamma_0)$ is outside the triangle if $\beta_0 < 0$.

   A. Skip Question
   B. only (i) is true
   C. only (ii) is true
   D. only (iii) is true
   E. only (i) and (ii) are true
   F. only (i) and (iii) are true
   G. only (ii) and (iii) are true
   H. (i), (ii) and (iii) are true

Shading

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7. (1 point) In a theatre there are two overhead lamps (consider point light sources). Lamp 1 has a fixed coloured lamp that emits light in the intensity $I_1 = (0.5, 1.0, 0.8)$, but Lamp 2 can be configured to emit any desired RGB intensity. The first lamp is at position $L_1 = (-\frac{\sqrt{3}}{2}, \frac{1}{2}, 1)$ and the second at position $L_2 = (\frac{\sqrt{3}}{2}, \frac{1}{2}, 1)$. A table is positioned on stage with a completely diffuse tablecloth. The material properties of the tablecloth is as follows: $k_a = (0, 0, 0); k_d = (0.4, 0.4, 0.5); k_s = (0, 0, 0)$. Consider the tabletop to be parallel to the ground (with constant y coordinate). How should we configure the intensity of the second lamp in order for the reflected light (perceived colour) to be $(0.2, 0.4, 0.3)$ at point $(0, 0, 1)$ on the tablecloth?

   A. Skip Question

   B. $(0.5, 1.0, 0.8)$ C. $(0.5, 0.0, 0.2)$ D. $(0.5, 1.0, 0.4)$

   E. $(0.6, 0.8, 0.2)$ F. $(0.4, 0.7, 0.3)$ G. $(0.6, 0.9, 0.5)$ H. $(0.4, 0.9, 0.5)$

8. (1 point) Consider two triangles parallel to the image plane. Triangle A is drawn after Triangle B. Triangle A has RGBA values $c_a = (0.6, 0.5, 0.9, 0.5)$ while Triangle B has RGBA values $c_b = (0.3, 0.9, 0.1, 0.3)$. The background has colour $bg = (0.5, 0.5, 0.5)$. What is the RGB color in the region where the projections of the two triangles overlap? Consider both are fully inside the view frustum and that alpha blending is enabled.

   A. Skip Question

   B. $(0.89, 1.0, 0.98)$ C. $(0.74, 0.87, 0.93)$ D. $(0.15, 0.235, 0.24)$

   E. $(0.52, 0.56, 0.64)$ F. $(0.21, 0.285, 0.075)$ G. $(0.48, 0.52, 0.62)$ H. $(0.3, 0.25, 0.45)$

9. ($1/2$ point) Which of the following statements are true when using alpha blending on a black background?

   i. If all triangles have the same alpha value but differing RGB values, the image will be the same independent of the drawing order.
   ii. If all triangles have the same RGB values but differing alpha values, the image will be independent of the drawing order.
   iii. A triangle with an alpha value of 0 does not affect the final image.

A. Skip Question
B. only (i) is true
C. only (ii) is true
D. only (iii) is true
E. only (i) and (ii) are true
F. only (i) and (iii) are true
G. only (ii) and (iii) are true
H. (i), (ii) and (iii) are true

Images

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10. ($1/2$ point) Given the index 33450 to access a pixel of an image loaded from a PPM file, calculate the corresponding pixel coordinates (i, j) based on the following header of the image file.

```
P3
550 475
5
```

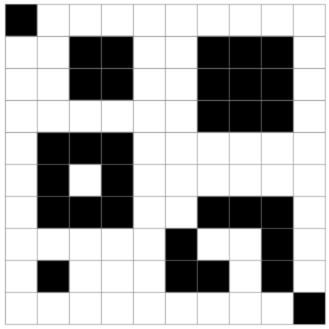
A. Skip Question

B. $(100, 160)$ C. $(150, 20)$ D. $(112, 47)$ E. $(15, 95)$ F. $(40, 95)$ G. $(67, 135)$ H. $(63, 125)$

11. (1 point) You are given the code to apply an image filter to an image pixel (i, j), the 3×3 filter and the binary 10×10 input image. If you apply the filter to all pixels in range $[1, \text{width} - 2] \times [1, \text{height} - 2]$, which of the following images corresponds to the correct output? Note that in the options below the border is grey to indicate pixels that are not evaluated by the filtering process (should not be taken into consideration in your analysis).

$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
$\frac{1}{8}$	0	$\frac{1}{8}$
$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$

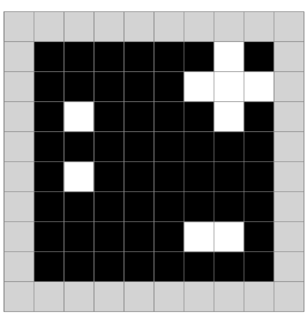
filter

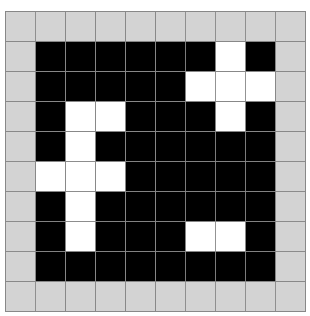


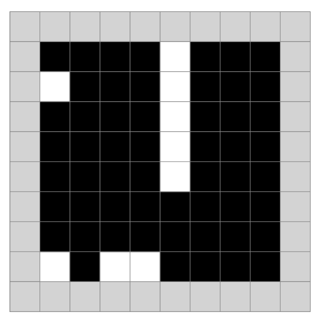
input image

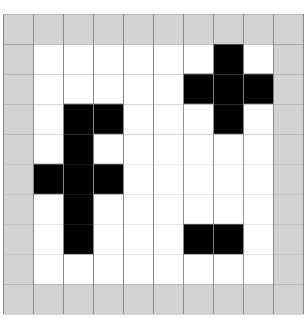
```
float Filter (Image & source, Image & filter, int i, int j) {
    float sum = 0;
    for (int x=0;x<filter.w;++x) {
        for (int y=0;y<filter.h;++y) {
            sum += filter.pixel(x,y) * source.pixel(i+x-filter.w/2,j+y-filter.h/2);
        }
    }
    if (sum <= 0.5 )
        return 0.0;
    else
        return 1.0;
};
```

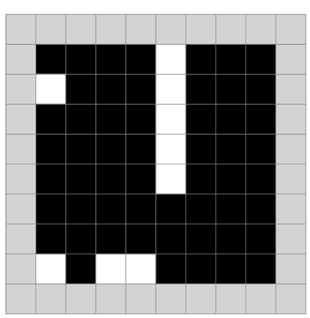
A. Skip Question

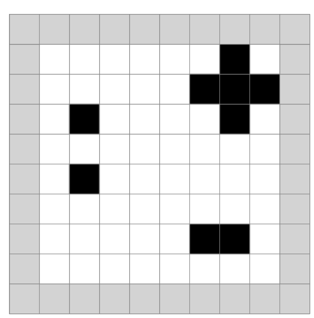
B. 

C. 

D. 

E. 

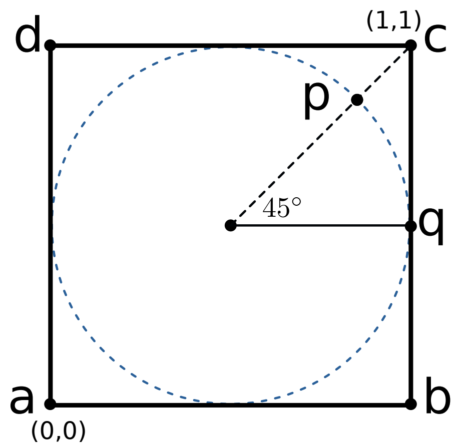
F. 

G. 

Textures

12. (1 point) Given colours at pixel positions $\{a, b, c, d\}$ we want to use bilinear interpolation to compute the colour of any point inside the square region defined by the four pixel centres, as shown in the image. Considering point p on the inscribed circle in the square, what would be the weight relative to pixel b when computing the colour at point p?

    Note that point p can be found by rotating point $q = (1.0, 0.5)$ by an angle of 45° counter-clockwise around the circle's centre.



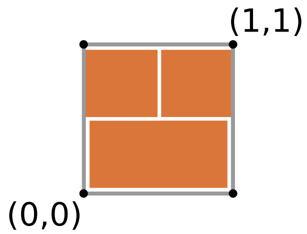
A. Skip Question B. $\frac{1}{16}$ C. $\frac{2}{16}$ D. $\frac{3}{16}$ E. $\frac{4}{16}$ F. $\frac{6}{16}$ G. $\frac{9}{16}$ H. $\frac{12}{16}$

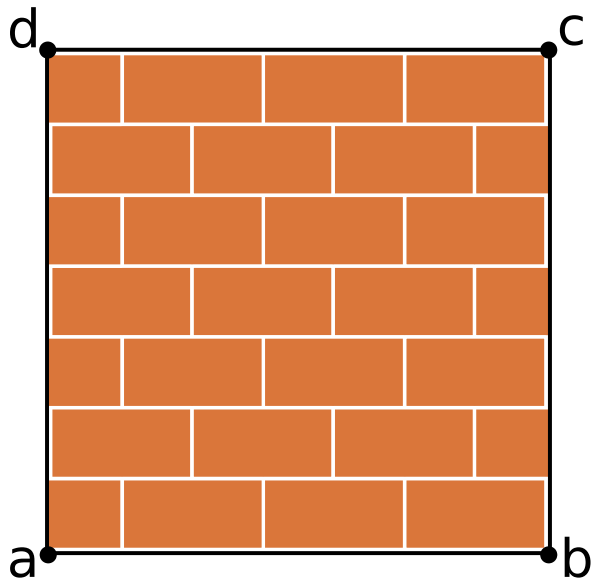
13. (½ point) Suppose you want to create a new mipmap method. Instead of dividing each dimension by 2 at each new level, you divide it by 5. Given a texture with dimensions 625x625, which of the following statements are true?

    i. The mipmap chain (excluding the original texture) has 4 levels.
    ii. Changing the texture including its mipmap from RGB to RGBA increases memory consumption by 1/5.
    iii. The memory consumption of level i is 25 times larger than level $i + 1$.

A. Skip Question
B. only (i) is true
C. only (ii) is true
D. only (iii) is true
E. only (i) and (ii) are true
F. only (i) and (iii) are true
G. only (ii) and (iii) are true
H. (i), (ii) and (iii) are true

14. (1 point) Given the texture on the left below and that corner vertex c has texture coordinates $(2.5, 2.0)$, what should be the texture coordinates of corner vertices $\{a, b, d\}$ to produce the quad as in the image on the right? Assume **repeat** mode is used.





A. Skip Question

B. $a = (0.0, 0.0); b = (2.5, 0.0); d = (0.0, 2.0)$

C. $a = (-1.0, -1.0); b = (2.5, 0.0); d = (-1.0, 2.0)$

D. $a = (-1.0, -1.5); b = (2.5, -1.5); d = (-1.0, 2.0)$

E. $a = (-0.5, -1.5); b = (1.5, -1.5); d = (-0.5, 2.0)$

F. $a = (-0.5, -0.5); b = (2.0, -0.5); d = (-0.5, 2.0)$

G. $a = (0, -0.5); b = (2.0, -0.5); d = (2.0, 0.5)$

H. $a = (-1.0, -0.5); b = (2.0, -0.5); d = (2.0, 0.5)$

Shadows

15. (1 point) We want to calculate shadows in the scene based on the shadow map below. Given a scene point, let us assume its corresponding texel location, which we calculate for shadow mapping, results in texture coordinates (i, j) and a depth of 0.3. Instead of only applying shadow mapping at (i, j), we instead calculate the average result of evaluating the shadow map in a 3×3 texel window centred at the texel containing (i, j). For which of the following texture coordinates should the light energy arriving at the scene point be scaled by $\frac{2}{3}$ to take the computed shadow correctly into account?

(1,1)

0.2	1	0.4	0.2	1	0.2	0.4	0.2	1	0.2
1	0.2	0.2	1	0.2	1	0.2	1	0.2	1
1	0.2	0.4	0.2	0.1	1	0.4	0.2	0.1	1
1	1	1	1	1	1	1	1	1	1
1	0.2	0.5	0.5	0.4	0.2	0.5	0.5	0.2	0.2

(0,0) (1,0)

A. Skip Question
B. $(0.15, 0.7)$ C. $(0.45, 0.3)$ D. $(0.25, 0.5)$ E. $(0.65, 0.5)$ F. $(0.85, 0.3)$ G. $(0.85, 0.7)$