Computer Graphics - CSE2215 - 2020/2021

Midterm Examination

1. i. Given the following projection matrix $\mathbf{M}$, what are the coordinates of point $\mathbf{P}$ in homogeneous coordinates such that $\mathbf{MP} = (0.5, 0.5, 1)$?

$$\mathbf{M} = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

A. $\mathbf{P} = (1, -1, 1, 1)$ B. $\mathbf{P} = (-1, -2, 8, 1)$ C. $\mathbf{P} = (\frac{1}{8}, \frac{1}{4}, 1, 1)$ D. $\mathbf{P} = (\frac{1}{8}, \frac{1}{4}, 2, 1)$

ii. Same question for $\mathbf{MP} = (-2, 1, 1)$ and

$$\mathbf{M} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -3 & 0 \end{bmatrix}$$

Solution: $\mathbf{P} = (4, -6, 2, 1)$

iii. Same question for $\mathbf{MP} = (-1, 0.5, 1)$ and

$$\mathbf{M} = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 0 \end{bmatrix}$$

Solution: $\mathbf{P} = (2, -5, 5, 1)$

2. i. Which sequence of transformations below when applied to a point is described by the following matrix M?

$$\mathbf{M} = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} & \sqrt{2}+1 \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} & \sqrt{2}-1 \\ 0 & 0 & 1 \end{bmatrix}$$

A. Translate by $(2, -\sqrt{2})$ then Rotate by 45^o
B. Rotate by 60^o then Translate by $(2, -2)$
C. Rotate by 60^o then Translate by $(\sqrt{2}+1, \sqrt{2}-1)$
D. Translate by $(\sqrt{2}+1, \sqrt{2}-1)$ then Rotate by 45^o

ii. Same question for the following matrix M:

$$\mathbf{M} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & 2 \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

Solution: Rotate by 60^o then Translate by $(2, -2)$

iii. Same question for the following matrix M:

$$\mathbf{M} = \begin{bmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & \frac{\sqrt{3}}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

Solution: Rotate by 60^o then Translate by $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$
or
Translate by $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ then Rotate by 60^o

3. i. Consider a camera centred at the origin and pointed in the *positive* z-axis direction, and a square centred at $(0, 0, 5)$ and parallel to the image plane. Initially, the focal length of the camera is set to $f = 2$. If we want to double the projected area of the square in the image plane, to what value should we set the focal length? Please assume that the square, even when increased, fits entirely onto the screen.

A. $\sqrt{2}$ B. 2 C. $2\sqrt{2}$ D. 4

ii. Same question for square centred at $(0, 0, 7)$, $f = 3$, and we want to triple the projected area.

Solution: $3\sqrt{3}$

iii. Same question for square centred at $(0, 0, 3)$, $f = 2$, and we want to halve the projected area of the square.

Solution: $\sqrt{2}$

4. i. A photographer wants to compose an image of two people standing on a large staircase, but due to COVID restrictions, they cannot stand side-by-side. He places his camera at position $(0, 0, 0)$ pointing at the *positive* z-axis direction and sets the focal length so the image plane is located at $z = 2$. He then asks one person to stand at position $p_1 = (15, 45, 30)$ and the second person at a lower stair, at position $p_2 = (x, 10, 20)$. If the photographer wants their projected positions to be apart by a distance of $d = \sqrt{13}$ on the image plane, which of the following values of x satisfies the required distance on the image plane?

A. X=-25 B. X=-10 C. X=10 D. X=40

ii. Same question for $z = 3$, $p_1 = (20, 30, 10)$, $p_2 = (x, 60, 30)$, and $d = \sqrt{10}$.

Solution: $x = 50$ (there is another possible solution for x, but was not a choice in the exam)

iii. Same question for $z = 4$, $p_1 = (25, 30, 20)$, $p_2 = (x, 20, 10)$, and $d = \sqrt{5}$.

Solution: $x = 15$ (there is another possible solution for x, but was not a choice in the exam)

5. Given the following projection matrix $\mathbf{P}$ we ask you to:

$$\mathbf{P} = \begin{bmatrix} \frac{1}{aspect} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2}{near-far} & \frac{far+near}{near-far} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

i. Explain how points are transformed using this matrix

> **Solution:** X is only scaled by aspect ration, Y and W coordinates are not affected by this matrix. Z coordinate is scaled by $\frac{2}{near-far}$ and translated by $\frac{far+\frac{near}{}}{near-far}$. Note that objects are not scaled depending on their Z value, when $aspect = 1$ the X and Y coordinates are projected directly to the screen. If $w = 1$ points inside the frustum are mapped to a cube with dimensions $[-1, +1]$.

ii. For this special projection matrix, if you project a point that is located on the near plane, what is its z-coordinate after projection? Same question for a point on the far plane.

> **Solution:** point $(0, 0, -near, 1)$ is projected to $\frac{-2near}{near-far} + \frac{far+near}{near-far} = -1$
> point $(0, 0, -far, 1)$ is projected to $\frac{-2far}{near-far} + \frac{far+near}{near-far} = +1$

iii. Does the perspective division have the same effect as with the traditional projection matrix from the lectures? Why?

> **Solution:** No, because there is no division by $w = -z$, now w is a constant, typically $w = 1$. In fact, there is no perspective division. This matrix projects objects flat onto the image plane.

iv. Given $\mathbf{p}' = \mathbf{P}\mathbf{p}$, where $\mathbf{p}$ is a 3D point in homogeneous coordinates, and $\mathbf{p}'$ the transformed point by $\mathbf{P}$. Is the z coordinate of $\mathbf{p}'$ linear in regards to the $\mathbf{z}$ coordinate of $\mathbf{p}$? Explain why.

> **Solution:** Yes, it is linear. Again, since w is constant, there is no scaling by $\frac{1}{-z}$, thus the projected z will assume a linear form, such as $az + b$.

6. i. Consider a room illuminated by a lamp at position $l = (1, 3, 2)$. The lamp is small enough to be considered a point light source. In the room there is a specular billiard ball with radius $r = 1$ and centred at position $c = (2, 4, 0)$. Where should you stand in order to observe the maximal highlight intensity on the ball at the surface point where its normal vector points towards the direction $(0, 0, 1)$?

A. $(1, 3, 2)$ B. $(2, 3, 2)$ C. $(3, 3, 3)$ D. $(3, 5, 2)$

ii. Same question for $l = (1, 5, 4)$, $r = 2$, $c = (3, 3, 0)$.

> **Solution:** $(5, 1, 4)$

iii. Same question for $l = (0, 4, 4)$, $r = 1$, $c = (3, 1, 0)$.

> **Solution:** $(4, 0, 2)$

7. To add shadows to a scene with a point light using a ray tracer, one can launch an additional *shadow ray*. To do so, we first proceed as for standard raytracing and find the intersection point **P** of the scene through a given pixel's centre. For this intersection point, a shadow ray **R** is shot. **R** is directed towards the light **L** and its origin corresponds to **P** but is slightly offset towards **L**. If **R** intersects any geometry before reaching **L**, **P** is considered in shadow, else, **P** is considered lit.

   Please explain the reason behind the offset and how to implement it mathematically (shift by a distance Epsilon). Would you have suggestions on how to make the offset more robust than using a fixed Epsilon shift?

   Solution: The offset avoids self-shadowing, or auto-intersection. To shift the point we should translate it in the direction from the point to the light source by ϵ:

   $\mathbf{P}' = \mathbf{P} + \epsilon \frac{\mathbf{L}-\mathbf{P}}{|\mathbf{L}-\mathbf{P}|}$

   Suggestion: use the orientation of the surface at **P** to set ϵ. The smaller the angle between $\mathbf{L} - \mathbf{P}$ and the surface normal the greater the distance between $\mathbf{P}'$ and surface for the same ϵ. Hence, for larger angles ϵ should be increased, or vice-versa.

8. i. Given a triangle defined by the vertices $v_0 = (-1, -1), v_1 = (2, 0)$ and $v_2 = (2, 4)$, and respective RGB colours $C_0 = (0.9, 0, 0), C_1 = (0.6, 0.9, 0)$ and $C_2 = (0, 0.9, 0.3)$. Determine the interpolated colour at point $p = (1, 1)$ using the definition of barycentric coordinates from the lectures.

   A. $(0.3, 0.3, 0.1)$
   B. $(0.5, 0.6, 0.1)$
   C. $(0.9, 0.9, 0.3)$
   D. $(0.5, 0.5, 0.3)$

   ii. Same question for $v_0 = (2, 0), v_1 = (6, 1), v_2 = (2, 5)$, colours $C_0 = (0.9, 0, 0), C_1 = (0.6, 0.9, 0), C_2 = (0, 0.9, 0.3)$, and $p = (4, 1)$

   Solution: $(0.48, 0.72, 0.09)$

   iii. Same question for $v_0 = (-1, 1), v_1 = (3, 1), v_2 = (2, 4)$, colours $C_0 = (0.9, 0, 0), C_1 = (0.6, 0.9, 0), C_2 = (0, 0.9, 0.3)$, and $p = (0, 2)$

   Solution: $(0.6, 0.3, 0.1)$

9. i. Consider a triangle T_1 with vertices $v_0 = (0, 0), v_1 = (4, 0), v_2 = (0, 4)$, and a second triangle T_2 with vertices $v_0 = (0, 0), v_1 = (4, 0), v_2 = (2, 2)$. If a point P has barycentric coordinates $(\alpha_1, \beta_1, \gamma_1)$ according to T_1, and the same point has barycentric coordinates $(\alpha_2, \beta_2, \gamma_2)$ according to T_2, then which of the following gives the correct relationship between the barycentric coordinates for any point P inside both triangles?

   A. $\alpha_2 = \alpha_1; \beta_2 \leq \beta_1; \gamma_2 = 2\gamma_1$
   B. $\alpha_2 \leq \alpha_1; \beta_2 \leq \beta_1; \gamma_2 \leq \gamma_1$
   C. $\alpha_2 \geq \alpha_1; \beta_2 = 2\beta_1; \gamma_2 = 2\gamma_1$
   D. $\alpha_2 = \alpha_1; \beta_2 \leq 0.5\beta_1; \gamma_2 \leq 0.5\gamma_1$

   ii. Same question for T_1 with vertices $v_0 = (0, 0), v_1 = (4, 0), v_2 = (0, 4)$, T_2 with vertices $v_0 = (0, 0), v_1 = (4, 0), v_2 = (0, 2$

Solution: $\alpha_2 \leq \alpha_1; \beta_2 = \beta_1; \gamma_2 = 2\gamma_1$

iii. Same question for T_1 with vertices $v_0 = (0,0), v_1 = (4,0), v_2 = (0,4)$, T_2 with vertices $v_0 = (0,0), v_1 = (2,0), v_2 = (0,4$

Solution: $\alpha_2 \leq \alpha_1; \beta_2 = 2\beta_1; \gamma_2 = \gamma_1$

10. Instead of a 3D pipeline, we are opting to generate a graphics pipeline that draws an axis-aligned rectangle on the screen, which is defined by its lower left and upper right corner vertex coordinates. Instead of a projection matrix, the vertices are directly expressed in 2D coordinates in the range $[-1,1] \times [-1,1]$, as they appear in the graphics pipeline after projection. As for the standard rasterization, pixels are filled based on their centre being covered. Imagine the image *Img* has width 10 and height 15 pixels and only stores two colours 0 and 1. The screen is initially entirely set to 0 and a drawn rectangle has the value 1.

For rasterization (filling in the image pixels), the following function is used:

```
void raster (int x, int y, int x2, int y2)
{
  for (int a=x; a<x2; ++a)
    for (int b=y; b>y2; --b)
    {
      Img.pixel(a,b)=1;
    }
}
```

Hint: Notice that b is decremented, as the image is stored top to bottom.

For the rectangle with corners $(-0.8, -0.2)$, $(0.55, 0.1)$, which would be the correct parameters for the function raster?

A. x=1, y=8, x2=8, y2=6
B. x=0, y=8, x2=8, y2=6
C. x=1, y=7, x2=8, y2=5
D. x=1, y=9, x2=8, y2=7

11. i. Consider an image with resolution of 2×2 pixels. You want to double the resolution to 4×4 and use bilinear interpolation to compute the colours of the new pixels. If the original image has colours $(1.0, 0.0, 0.0), (0.0, 0.0, 1.0), (0.0, 1.0, 0.0), (1.0, 0.0, 0.0)$ at pixels $(0,0), (1,0), (0,1), (1,1)$, respectively, what should be the colour of pixel $(2,2)$ in the new image?

A. $(\frac{5}{8}, \frac{3}{16}, \frac{3}{16})$
B. $(\frac{3}{4}, \frac{1}{4}, \frac{1}{4})$
C. $(\frac{9}{10}, \frac{5}{8}, \frac{5}{8})$
D. $(\frac{3}{4}, \frac{3}{16}, \frac{3}{16})$

ii. Same question for colours $(1.0, 0.0, 1.0), (0.0, 0.0, 1.0), (0.0, 1.0, 0.0), (1.0, 1.0, 0.0)$ and pixel $(1,2)$

Solution: $(\frac{3}{8}, \frac{3}{4}, \frac{1}{4})$

iii. Same question for colours $(0.0, 1.0, 0.0), (1.0, 0.0, 1.0), (1.0, 1.0, 1.0), (0.0, 0.0, 1.0)$ and pixel $(2,1)$

Solution: $(\frac{5}{8}, \frac{1}{4}, \frac{13}{16})$

12. Prove that for bilinear texture interpolation, if you first linearly interpolate along the x axis and then along the y axis, you obtain the same result as first linearly interpolating along the y axis followed by the x axis. Examples may support your explanation but they do not count as proof.

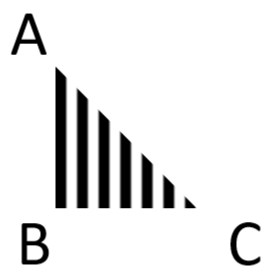
Solution: Suppose a square with corners A (top-left), B (top-right), C (bottom-left), and D (bottom-right), and weights α for the horizontal axis and β for the vertical axis.

Solution for x axis first:
$E = \alpha A + (1-\alpha)B$
$F = \alpha C + (1-\alpha)D$
$P_{hv} = \beta E + (1-\beta F) = \beta\alpha A + \beta(1-\alpha)B + (1-\beta)\alpha C + (1-\beta)(1-\alpha)D$

Solution for y axis first:
$G = \beta A + (1-\beta)C$
$H = \beta B + (1-\beta)D$
$P_{vh} = \alpha G + (1-\alpha)H = \alpha\beta A + \alpha(1-\beta)C + (1-\alpha)\beta B + (1-\alpha)(1-\beta)D$

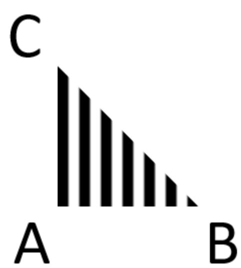
Hence, $P_{hv} = P_{vh}$

13. i. Given this black/white triangle, what **minimally-sized** texture can be used with what texture coordinates on A,B,C to obtain this result? What interpolation method and border behaviour to choose?



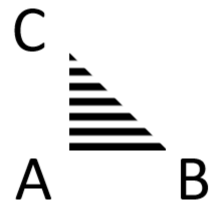
Solution:
Texture size: 2x1 with pixel (0,0) black and pixel (1,0) white
Coordinates: A(0,0) or (0,1), B(0, 0), C(6.5,0)
Interpolation: nearest-neightbors
Border behaviour: x-axis repeat mode and y-axis clamp mode

ii. Same question for following image



Solution:
Texture size: 2x1 with pixel (0,0) black and pixel (1,0) white
Coordinates: A(0,0), B(6.5, 0), C(0,0) or (0,1)
Interpolation: nearest-neightbors
Border behaviour: x-axis repeat mode and y-axis clamp mode

iii. Same question for following image



Solution:
Texture size: 1x2 with pixel (0,0) black and pixel (0,1) white
Coordinates: A(0,0), B(0, 0), C(0,6.5)
Interpolation: nearest-neightbors
Border behaviour: x-axis clamp mode and y-axis repeat mode