Exam TI2710-A/IN2405-A

January 25th 2012

Question 1 - LTI systems

(a) (1 p.) Give an example of a causal LTI system and explain why this system is causal.

Consider the following input-output relation:

$$y[n] = 2x[n+3] + n.$$

(b) (2 p.) Show whether or not this system is linear.

(c) (1 p.) Show whether or not this system is time-invariant.

Given the impulse response of an LTI system $\mathcal{S}_1$:

$$h_1[n] = \frac{1}{3}\left(\delta[n] + \delta[n-1] + \delta[n-2]\right).$$

The input to this system is given by $x_1[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$.

(d) (2 p.) Compute the output $y_1[n]$ of this system for input $x_1[n]$ by explicitly computing the convolution.

(e) (2 p.) Show by using the super-position principle, that $y_1[n]$ can also be written as $y_1[n] = ah_1[n] + bh_1[n-1] + ch_1[n-2]$ and determine the values of a, b and c.

System $\mathcal{S}_1$ is put in cascade with a system $\mathcal{S}_2$ with the following input-output relation

$$y_2[n] = x_2[n] - x_2[n-2].$$

(f) (2 p.) Compute the impulse response of the cascaded system.

Question 2 - Sampling and Fourier Series

A continuous-time signal is defined by

$$y(t) = 5 + 2\cos(20\pi t + \pi/2).$$

(a) (1 p.) Make a sketch of the signal $y(t)$ and show that the period of $y(t)$ equals $T_0 = 0.1$ sec.

(b) (1 p.) Suppose that this signal will be sampled with sampling frequency $f_s = 15$ Hz, leading to the sampled signal $y[n]$. Plot at least 15 samples of the sampled signal $y[n]$.

(c) (2 p.) Reconstruction of the time-continuous signal from the samples of $y[n]$ will be done based on an interpolation with sinc functions. Explain whether or not (and why!) the reconstructed signal $\tilde{y}(t)$ becomes identical to the original $y(t)$.

The Fourier integral of $y(t)$ is given by

$$a_k = \frac{1}{T_0}\int_0^{T_0} y(t)e^{-j(2\pi/T_0)kt}dt.$$

(d1) (1 p.) Show that $a_1 = e^{\frac{\pi}{2}j}$.

(d2) (1 p.) Show that $a_2 = 0$.

(d3) (1 p.) Compute a_{-2}, a_{-1} and a_0.

(e) (2 p.) Explain why the Fourier coefficients a_k for signal $y(t)$ are zero for $|k| \geq 2$.

(f) (1 p.) Sketch the complex spectrum of signal $y(t)$. Clearly mark the frequency and amplitude axes.

Question 3 - Filtering

A linear time-invariant system $\mathcal{S}_1$ is described by the following impulse response:

$$h_1[n] = 4\delta[n] + 4\delta[n-2].$$

(a) (2 p.) Show that the frequency response $H_1(e^{j\hat{\omega}})$ can be written as

$$H_1(e^{j\hat{\omega}}) = 8e^{-j\hat{\omega}} \left(\cos\left(\hat{\omega}\right)\right).$$

(b) (1 p.) Sketch the magnitude response of $H_1(e^{j\hat{\omega}})$. Clearly mark the zeros and local maxima in your plot.

(c) (2 p.) Sketch the principal value of the phase response of $H_1(e^{j\hat{\omega}})$. (Principal value means that all values of $\arg H_1(e^{j\hat{\omega}})$ are between $-\pi$ and $+\pi$.)

Let the input $x_1[n]$ to system $\mathcal{S}_1$ be given by

$$x_1[n] = \cos\left(\frac{\pi}{3}n + \frac{\pi}{3}\right).$$

(d) (2 p.) Use the magnitude and phase of $H_1(e^{j\hat{\omega}})$ to determine the output $y_1[n]$ of system $\mathcal{S}_1$ for input $x_1[n]$.
(*Hint:* $\cos(0) = 1$, $\cos(\pi/6) = \frac{1}{2}\sqrt{3}$, $\cos(\pi/4) = \frac{1}{2}\sqrt{2}$, $\cos(\pi/3) = \frac{1}{2}$).

System $\mathcal{S}_1$ is put in cascade with a system $\mathcal{S}_2$ with impulse response

$$h_2[n] = \frac{1}{2}h_1[n].$$

(e) (2 p.) Give the frequency response $H(e^{j\hat{\omega}})$ of the cascaded system.

(f) (1 p.) Explain whether this cascaded system is an FIR or IIR filter.

Question 4 - IIR Filters

Let $a \in \mathbb{R}$ be a constant. Consider the input-output relation of the following linear time-invariant system that is initially at rest:

$$y[n] = ay[n-2] + 2x[n].$$

(a) (2 p.) Determine the first 6 values of the impulse response of this system.

(b) (2 p.) For which values of a is this system stable?

Let the input to this system be given by

$$x[n] = 2\delta[n] + \delta[n-1] + 2\delta[n-2].$$

(c) (2 p.) Determine the output signal $y[n]$ in terms of the impulse response $h[n]$.

(d) (2 p.) Compute the frequency response $H(e^{j\hat{\omega}})$ of this system by using the given input-output relation.

(e) (2 p.) Explain in words how the impulse response can be computed directly from the frequency response $H(e^{j\hat{\omega}})$ of this system (it is not necessary to perform this computation).