Amswers Mid-term Exam Signal Processing TI2716-A

September 25^{th}, 2014
09:00 - 11:00 h

Question 1 (5 points total)

(a) (1 p.) The system is causal, as for any n, only values of $x[m]$ with $m \leq n$ are used. In a non-causal system, values of $x[m]$ would also be used with $m > n$.

(b) (2 p.) When delaying $y[n]$ by n_0, we get $y[n-n_0] = x[n-n_0] - 5x[n-n_0-1] + 1$. When delaying $x[n]$ by n_0 before computing $y[n]$, the output would be $y[n-n_0] = x[n-n_0] - 5x[n-n_0-1] + 1$. As these both outcomes are identical, the system is time-invariant.

(c) (2 p.) Take as input a signal $x[n] = \alpha x_A[n] + \beta x_B[n]$. If the system is linear, the corresponding outcome $y[n]$ should equal $\alpha y_A[n] + \beta y_B[n]$.
$y[n] = (\alpha x_A[n] + \beta x_B[n]) - 5(\alpha x_A[n-1] + \beta x_B[n-1]) + 1 \neq \alpha y_A[n] + \beta y_B[n]$
so the system is not linear.

Question 2 (9 points total)

(a) (1 p.) $y[n] = x[n] - 2x[n-1]$

(b) (1 p.) S_1 is a first-older filter.

(c) (1 p.) $x_1[n] = \delta[n] + 2\delta[n-1] + 2\delta[n-2] + \delta[n-3]$.

(d) (2 p.) We need to compute $h_1[n] * x_1[n]$.

$$\begin{bmatrix}1 & -2\end{bmatrix}\begin{bmatrix}1 & 2 & 2 & 1 & 0\\ 0 & 1 & 2 & 2 & 1\end{bmatrix} = \begin{bmatrix}1 & 0 & -2 & -3 & -2\end{bmatrix}$$

so the outcome is

n	≤ -2	-1	0	1	2	3	4	≥ 5
$x[n]$	0	0	1	0	-2	-3	-2	0

(e) (2 p.) $h[n] = h_1[n] * h_2[n] = 2\delta[n] - 4\delta[n-1] + \delta[n-2] + 2\delta[n-3]$.

(f) (2 p.) Convolve $h[n]$ with signal $x_1[n]$. The result $y[n] =$

n	≤ -2	-1	0	1	2	3	4	5	6	≥ 7
$x[n]$	0	0	2	0	-3	-6	-6	-3	-2	0

Question 3 (7 points total)

(a) (2 p.) The signal period is 0.005 sec. The maximum value is reached at $t = 0.00125$.
(b)

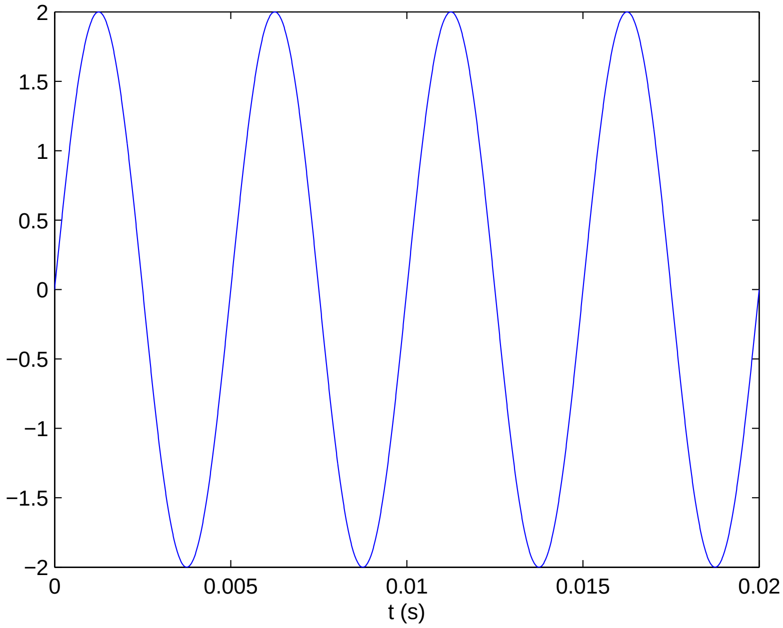

Figure 1: The plot for $x(t) = 2\cos(400\pi t - \pi/2)$.

(b) (1 p.) The Nyquist sampling rate is 400 Hz.

(c) (2 p.) $x[n] = 2\cos(\pi/2n - \pi/2)$

(d) (2 p.) The time between consecutive sample values is now assumed to be $1/200 = 0.005s$. The signal period is 4 samples in the discrete domain, so will now be $0.02s$. That means the cosine's frequency will be 50 Hz. The reconstructed $x(t)$ will thus sound lower than the original $x(t)$.

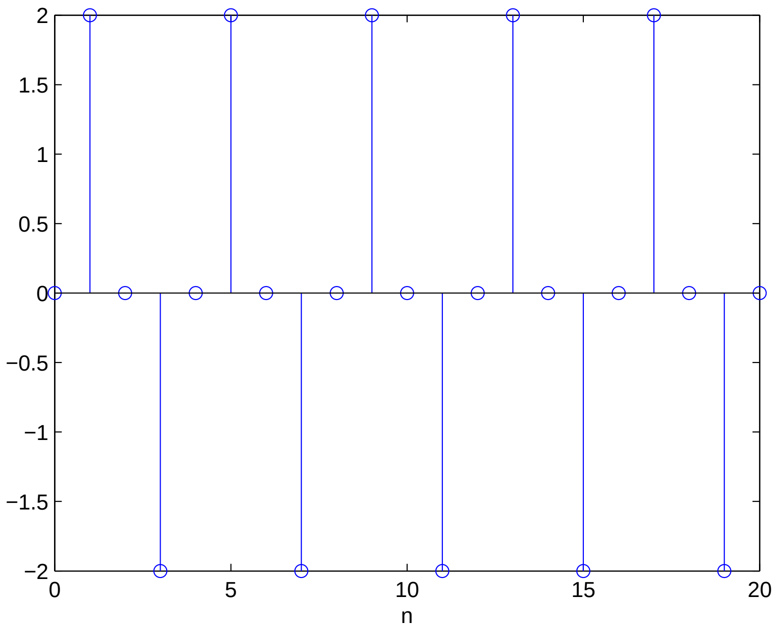

Figure 2: The plot for $x[n] = 2\cos(\pi/2n - \pi/2)$.

Question 4 (6 points total)

(a) (1 p.) $z = 4(\cos(\frac{4}{3}\pi) + j\sin(\frac{4}{3}\pi))$. (Note: this may have been the question on which most mistakes were made. Many students forgot that the number is in the 3rd quadrant, and that therefore, one should add π to the angle.)

(b) (1 p.) $z = 4e^(j\frac{4}{3}\pi)$

(c) (1 p.) $w = 2(\cos(\frac{\pi}{2}) + j\sin(\frac{\pi}{2}))) = 2j$.

(d) (1 p.)

(d (e)) (2 p.) $(-2 - 2\sqrt(3)j)(2j) = 4\sqrt(3) - 4j = 8e^(j\frac{11}{6}\pi)$

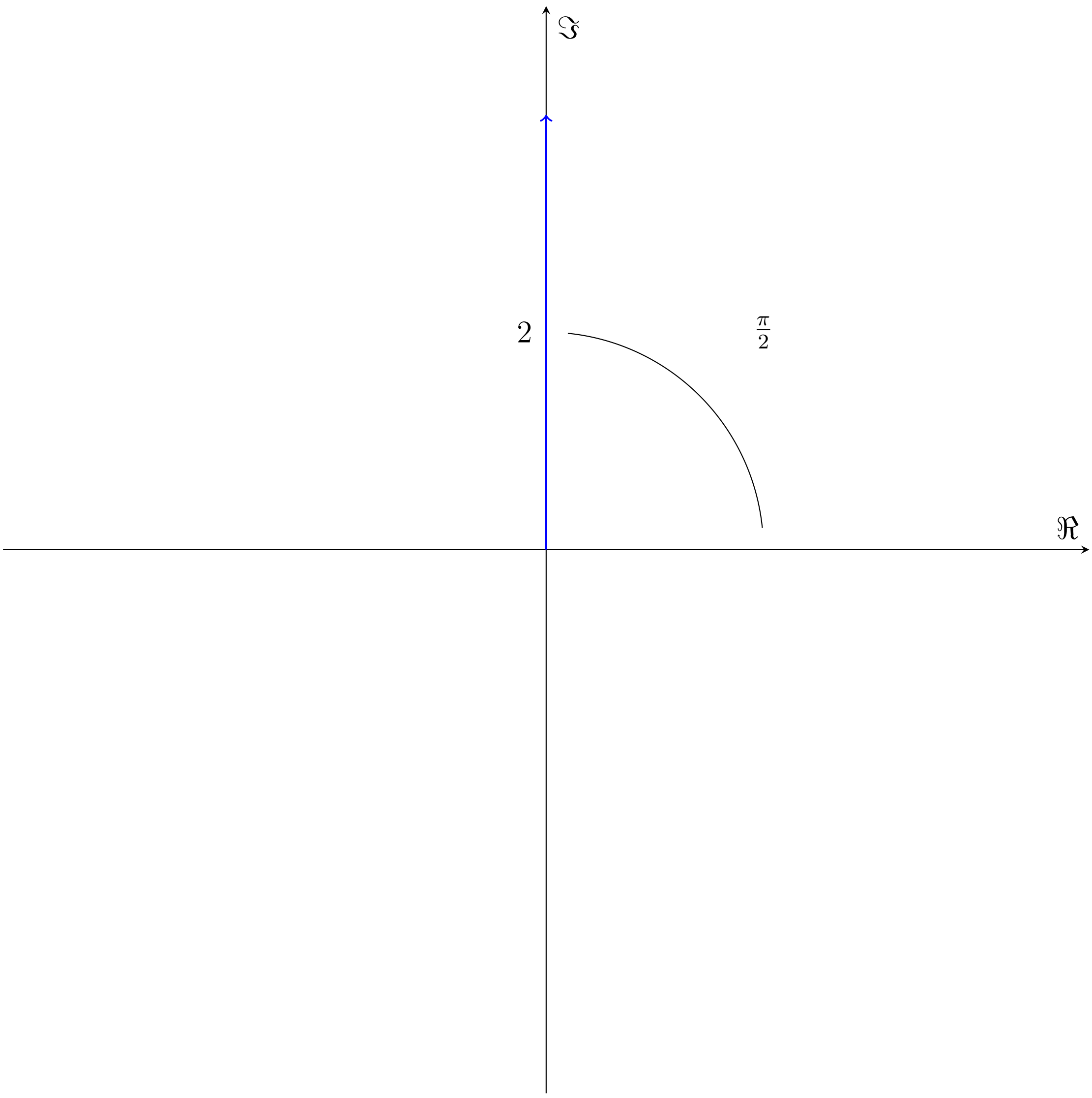

Figure 3: w on the complex plane

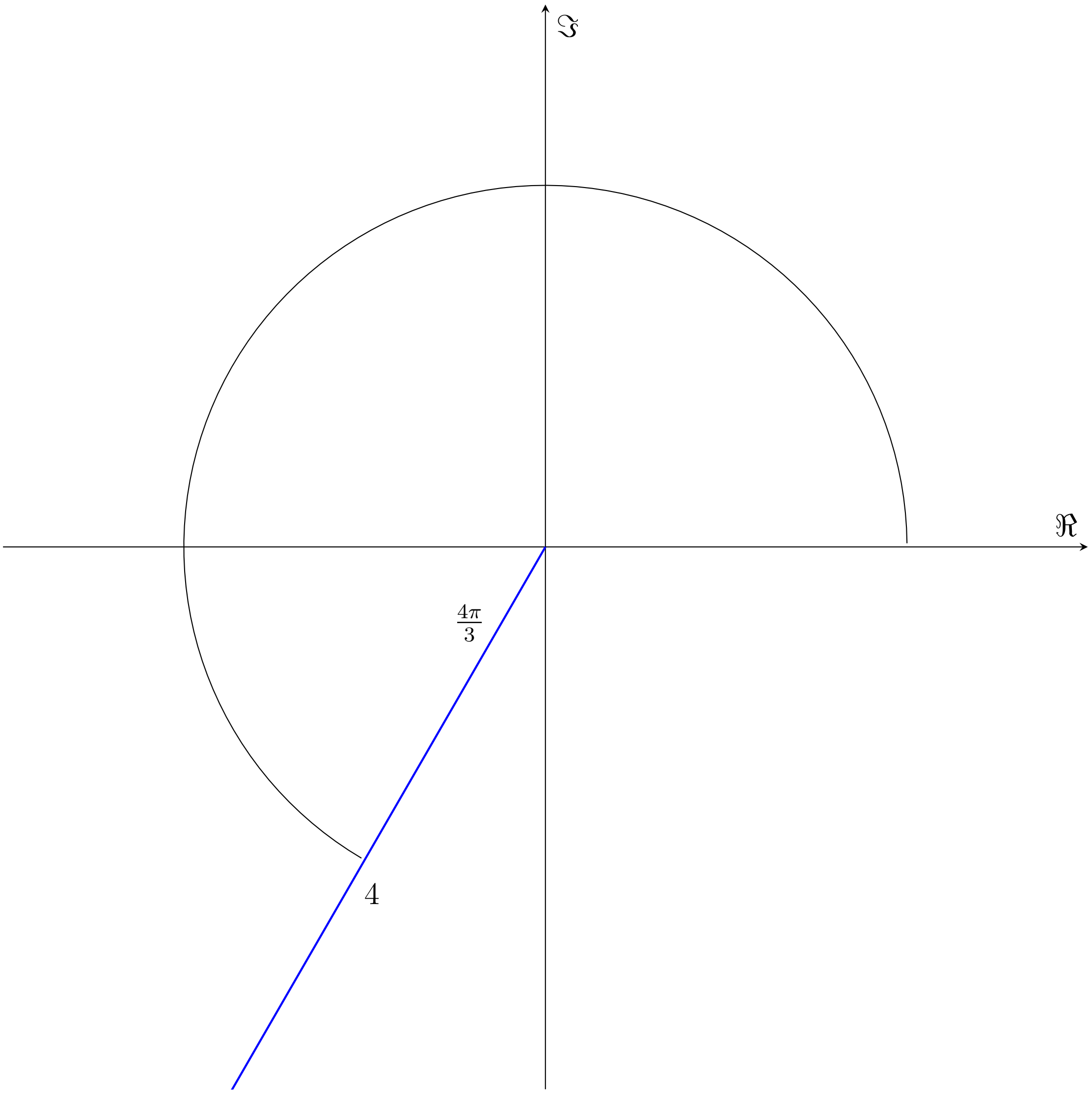

Figure 4: z on the complex plane