Signal Processing CSE2220
Partial exam 1
October 3, 2019, 09.00-11.00 h

- This exam has 4 questions, for which a total of 23 points can be obtained.
- The allotted time for this exam is 2 hours.
- Use of the Equation Sheet CSE2220 is permitted. On the equation sheet, hand-written notes are permitted in the page margins and on one blank back of a printed page.
- Use of a calculator is not permitted.
- For each of the questions, start your answer on a new page.
- In case you do not answer a question, please still explicitly list the question on your exam paper, and clearly indicate you did not answer it (e.g. by leaving the rest of the line blank, or by a ‘-’ after the question number).
- Questions should be answered in English.

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Question 1 (6 points)

The fundamental frequencies in human speech range between 85 Hz (for male adult voices) to 500 Hz (for childrens' voices).

We want to build a digital system that processes human speech. For this, we need to choose an appropriate sampling frequency.

a) (1 pt) Based on the information above, discuss what sampling frequency you would choose for the speech processing system.

We consider a continuous-time signal $x_1(t)$, which is defined as follows:

$$x_1(t) = 4\cos\left(50\pi t - \frac{1}{2}\pi\right) + 4\cos\left(10\pi t + \pi\right).$$

b) (1 pt) Determine the fundamental frequency of $x_1(t)$.

We sample signal $x_1(t)$ with sampling frequency $F_s = 70$ Hz, yielding a discrete-time signal $x_1[n]$.

c) (1 pt) Give the expression of signal $x_1[n]$.

We consider a pure cosine signal $x_2(t)$, which is defined as follows:

$$x_2(t) = 2\cos\left(300\pi t - \frac{\pi}{2}\right).$$

Signal $x_2(t)$ is sampled with a sampling frequency $F_s = 600$ Hz, yielding a discrete-time signal $x_2[n]$.

d) (1 pt) Make a clear plot of signal $x_2[n]$ for $0 \leq n \leq 8$.

We now consider a continuous-time signal $x_3(t)$, which is defined as follows:

$$x_3(t) = 5 + \cos\left(50\pi t + \frac{\pi}{2}\right) + \cos\left(-50\pi t - \frac{\pi}{2}\right) - \cos\left(50\pi t - \frac{\pi}{2}\right).$$

e) (1 pt) Rewrite $x_3(t)$ in the form $x(t) = c + A\cos(\omega t + \phi)$.

f) (1 pt) Make a clear plot of the signal $x_3(t)$ for $0 \leq t \leq 0.2$ seconds.

Question 2 (6 points)

We consider a digital input signal $x_4[n]$, specified as:

n	< 0	0	1	2	3	4	5
$x_4[n]$	0	5	0	0	-3	2	1

a) **(1 pt)** Rewrite $x_4[n]$ as a sum of impulse signals $\delta[n]$.

We consider a system S_A with input-output relation $y_A[n]$, specified as follows:

$$y_A[n] = x[n-2] + x[n].$$

b) (1 pt) Prove that system S_A is linear.

c) (1 pt) Prove that system S_A is time-invariant.

d) (1 pt) Determine the impulse response $h_A[n]$ for system S_A.

e) (1 pt) Signal $x_4[n]$ is given as input to system S_A, yielding an output signal $y_4[n]$. Calculate $y_4[n]$ by applying direct convolution.

Consider a sensor, that measures the temperature of a room every minute. The sensor can measure temperatures between -50 and +60 degrees Celsius. However, the measurements are noisy, and expected deviations from the true temperature range between -1 and +1 degree Celsius. We apply a filter to get a better estimate of the true temperature throughout the day. For this filter, you have the choice between four FIR filters with the following impulse responses:

- FIR1: $h[n] = \frac{1}{2}\delta[n] + \frac{1}{2}\delta[n-1]$;
- FIR2: $h[n] = \frac{1}{30}\sum_{k=0}^{29}\delta[n-k]$;
- FIR3: $h[n] = \delta[n] - \delta[n-1]$;
- FIR4: $h[n] = \delta[n] - \delta[n-30]$.

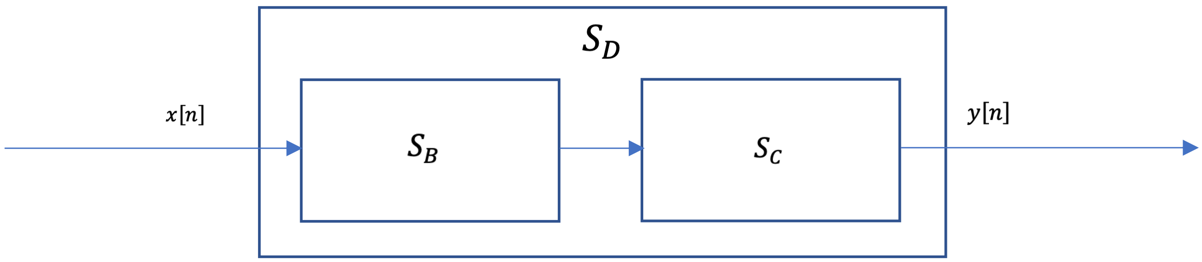
f) (1 pt) Which of the above FIR filters would you apply to the temperature sensor? Explain your answer in no more than five sentences.

Question 3 (5 points)

We consider a system S_B with the following impulse response $h_B[n]$:

$$h_B[n] = 3\delta[n] + 5\,\delta[n-2]\,.$$

System S_B is put in cascade with a system S_C. The result is a system S_D, as illustrated in the figure below:



S_D has the following impulse response $h_D[n]$:

$h_D[n] = -6\delta[n+1] - 10\,\delta[n-1]\,.$

- **a) (1 pt)** Give the input-output relation $y_C[n]$ of system S_C.
- **b) (1 pt)** Explain whether system S_C is causal.

We consider an input signal $x_5[n]$:

$$x_5[n] = 4\delta[n] - \frac{1}{2}\delta[n-1]\,.$$

Signal $x_5[n]$ is given as input to system system S_D, resulting in an output signal $y_5[n]$.

- **c) (1 pt)** Use the superposition principle to determine output signal $y_5[n]$.

We consider an IIR filter, of which the input-output relation $y_6[n]$ is defined as follows:

$$y_6[n] = 6y[n-1] + 2x[n]$$

- **d) (1 pt)** Explain whether the IIR filter above is stable.
- **e) (1 pt)** Determine the impulse response $h_6[n]$ for the IIR filter above.

Question 4 (6 points)

We consider a continuous-time signal $x_7(t)$, defined as:

$$x_7(t) = 4\cos(100\pi t) + 2\cos\left(100\pi t + \frac{\pi}{3}\right) + 2\cos(100\pi t + \pi)\,.$$

$x_7(t)$ can be rewritten in the form $x_7(t) = Xe^{j100\pi t} + X^* e^{-j100\pi t}$, where X denotes the complex amplitude.

a) **(1 pt)** Determine X.

b) **(1 pt)** Plot X in the complex plane.

Alternatively, $x_7(t)$ can be rewritten as a single cosine: $x_7(t) = A cos(100\pi t + \phi)$.

c) **(1 pt)** Determine A.

We consider a continuous-time signal $x_8(t)$, defined as:

$$x_8(t) = x_7(t) + 2 - 8\cos\left(200\pi t - \frac{\pi}{3}\right) + \cos(400\pi t)\,.$$

d) **(1 pt)** Determine the fundamental period T_0 of signal $x_8(t)$.

e) **(1 pt)** Make a clear plot of the amplitude spectrum of signal $x_8(t)$. Clearly indicate the variables and quantities in the axes.

f) **(1 pt)** Make a clear plot of the phase spectrum of the signal $x_8(t)$. Clearly indicate the variables and quantities in the axes.