Signal Processing CSE2220

Partial exam 2

November 6, 2019, 13.30-15.30 h

- This exam has 3 questions, for which a total of 25 points can be obtained.
- The allotted time for this exam is 2 hours.
- Use of the Equation Sheet CSE2220 is permitted. On the equation sheet, hand-written notes are permitted in the page margins and on one blank back of a printed page.
- Use of a calculator is not permitted.
- For each of the questions, start your answer at the top of a blank page.
- In case you do not answer a question, please still explicitly list the question on your exam paper, and clearly indicate you did not answer it (e.g. by leaving the rest of the line blank, or by a '-' after the question number).

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Question 1 (7 points)

Bats are associated with a few diseases that affect people, such as rabies and histoplasmosis. The frequencies of bat voice calls are in the range of 10 kHz -120 kHz. However, human hearing sensitivity only ranges from 20 – 20 kHz. As a consequence, humans may not actually hear a bat call. For this reason, we want to build a simple system that can detect bat calls with digital signal processing techniques.

The system uses an analogue microphone, recording the environmental sound as a continuous-time signal. For simplicity, we assume that if a signal recorded by the microphone contains frequencies in the range of 10 kHz - 120 kHz, these frequencies can only have originated from bat calls.

We first need to convert the continuous-time signal into a discrete-time signal. For this, a sampling frequency f_s needs to be chosen.

(a) **(1 pt)** What f_s should minimally be chosen, such that we avoid aliasing?

(b) **(2 pt)** For our system, we choose f_s = 400 KHz. If we do this, what is the interval of the digital frequency $\hat{\omega}$ (with $-\pi \leq \hat{\omega} \leq \pi$) in which we expect to detect bat calls?

To build our classifier we need to go through 2 steps.

1. Design a filter that will allow the frequency range of the bat to be analyzed.
2. Have a thresholding criterion to decide if the filtered signal contains a bat signal or not.

(c) **(2pt)** Sketch a filter that would satisfy the first step in our classifier with a clear indication of the values and the labels of the axis. What is the type of the filter chosen?

(d) **(2pt)** Choose a thresholding criterion that will satisfy point 2 in the steps. Explain why you chosen criterion method would work in solving the classification problem.

Question 2 (9 points)

An LTI system S_1 is specified by the following input-output relation:

$$y_1[n] = 6x[n-1] + 6x[n-2] + 6x[n-3].$$

(a) (1 pt) Calculate the impulse response $h_1[n]$ of system S_1 .

(b) (1 pt) Calculate the frequency response $H_1(e^{j\hat{\omega}})$ of system S_1 .

(c) (1 pt) Make a clear plot of the magnitude $|H_1(e^{j\hat{\omega}})|$ for $-\pi \leq \hat{\omega} \leq \pi$.

(d) (1 pt) Make a clear plot of the phase $\arg\left(H_1(e^{j\hat{\omega}})\right)$ for $-\pi \leq \hat{\omega} \leq \pi$.

An input signal $x_1[n]$ is defined as follows:

$$x_1[n] = 4\cos\left(\frac{\pi}{2}n\right) + 4 .$$

Signal $x_1[n]$ considers a pure cosine. As a consequence, within the interval $-\pi \leq \hat{\omega} \leq \pi$, $X_1(e^{j\hat{\omega}})$ will only be nonzero for a few dedicated digital frequencies $\hat{\omega}$.

(e) (1 pt) Make a clear plot of the magnitude $|X_1(e^{j\hat{\omega}})|$ for $-\pi \leq \hat{\omega} \leq \pi$.

Signal $x_1[n]$ is given as input to system S_1.

(f) (1 pt) Give an expression for the output $y_1[n]$.

$H_1(e^{j\hat{\omega}})$ is the Discrete-Time Fourier Transform (DTFT) of $h_1[n]$. In practice, we will work with Discrete Fourier Transforms (DFTs) instead. Say that we take the 4-point DFT of $h_1[n]$, yielding $H_1[k]$.

(g) (1 pt) Make a clear plot of the magnitude $|H_1[K]|$ by sampling the $|H_1(e^{j\hat{\omega}})|$ to get magnitude of the the 4-point DFT for $-\pi \leq \hat{\omega} \leq \pi$. Plot against the $\hat{\omega}$ not the K (x-axis should be the $\hat{\omega}$).

(h) (1 pt) Make a clear plot of the phase $\arg(H_1[K])$ by sampling the phase $\arg\left(H_1(e^{j\hat{\omega}})\right)$ to get the phase of the 4-point DFT for $-\pi \leq \hat{\omega} \leq \pi$. Plot against the $\hat{\omega}$ not the K (x-axis should be the $\hat{\omega}$).

Note: this question continues on the next page.

An LTI system S_2 is specified by the following input-output relation:

$$y_2[n] = 6x[n-2] + 6x[n-3] + 6x[n-4].$$

We compare the frequency response $H_2(e^{j\hat{\omega}})$ of system S_2 to the frequency response $H_1(e^{j\hat{\omega}})$ of system S_1.

(i) **(1 pt)** Explain whether the magnitude and phase of frequency response $H_2(e^{j\hat{\omega}})$ will be different from the magnitude and phase of frequency response $H_1(e^{j\hat{\omega}})$.

Question 3 (8 points)

We consider an 8-point discrete-time signal $x_2[n]$, which is specified as follows:

$$x_2[n] = 4\delta[n] + 3\delta[n-1] + 2\delta[n-2] + 3\delta[n-3] + 4\delta[n-4] + 3\delta[n-5] + 2\delta[n-6] + 3\delta[n-7].$$

We wish to obtain the 8-point DFT $X_2[k]$ for signal $x_2[n]$. Generally, an 8-point DFT can be expressed as a matrix multiplication:

$$\begin{bmatrix} X[0] \\ X[1] \\ \vdots \\ X[7] \end{bmatrix} = D_8 \begin{bmatrix} x[0] \\ x[1] \\ \vdots \\ x[7] \end{bmatrix}$$

where D_8 is the following 8×8 matrix:

$$D_8 = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j & j & -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j & -1 & -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j & -j & \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j & -j & \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j & -1 & \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j & -j & -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j & -j & \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j & -1 & \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j & -j & -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j & j & -\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}j & -1 & -\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j & -j & \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}j
\end{bmatrix}$$

(a) (2 pt) Give the full 8-point DFT $X_2[k]$ (with $k = 0, 1, \ldots, 7$) for signal $x_2[n]$.

Note: this question continues on the next page.

We pass signal $x_2[n]$ through an LTI system S_3, of which the impulse response is given by:

$$h_3[n] = \delta[n] + 2\delta[n-1] + \delta[n-2].$$

If we compute the output signal **using linear convolution**, we obtain an output signal $y_{L2}[n]$.

(b) (2 pt) Compute the output signal $y_{L2}[n]$ using linear convolution.

Instead of using linear convolution, we can alternatively obtain the output signal through **circular convolution**. For this, we first considering how $X_2[k]$ and $H_3[k]$ yield $Y_2[k]$, the DFT of the output signal in the frequency domain.

The first coefficients of the 8-point DFT $H_3[k]$ for $h_3[n]$ are given by:

k	0	1	2	3	4
$H_3[k]$	4	$2.4142 - 2.4142j$	$-2j$	$-0.4142 - 0.4142j$	0

(c) (1 pt) Calculate $Y_2[k]$, **m**aking use of the DFTs $X_2[k]$ and $H_3[k]$.

Using $Y_2[k]$, we can calculate the corresponding output signal in the (discrete) time domain. We call this signal $y_{C2}[n]$.

(d) (2 pt) Compute the signal $y_{C2}[n]$ from $Y_2[k]$.

The signals $y_{L2}[n]$ and $y_{C2}[n]$ are not exactly the same.

(e) (1 pt) Explain which signal samples in $y_{L2}[n]$ and $y_{C2}[n]$ are different and give the reason for this difference.