Signal Processing CSE2220

Resit exam

January 26, 2022, 13.30-16.30 h

- This exam has 5 questions, for which a total of 36 points can be obtained.
- The allotted time for this exam is 3 hours.
- Use of the Equation Sheet CSE2220 is permitted. On the equation sheet, hand-written notes are permitted in the page margins and on one blank back of a printed page.
- Use of a calculator is not permitted.
- For each of the questions, answer in the dedicated answer section of that question in the answer sheet.
- In case you do not answer a question, please still explicitly list the question on your exam paper, and clearly indicate you did not answer it (e.g. by leaving the rest of the line blank, or by a '-' after the question number).
- Questions should be answered in English.

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Question 1 (6 points)

A discrete-time system S_w is described by the following input-output relation:

$$S_w: \quad y[n] = x[n^2] + 3x[n-3] + \cos(n\pi).$$

(a) (1 pt) Show whether system S_w is linear or not.
(b) (1 pt) Show whether system S_w is time-invariant or not.

Given are two LTI systems S_t and S_r described as:

$$S_t: \quad y[n] = x[n] - x[n-1]$$

$$S_r: \quad y[n] = x[n] + 4x[n-1] - 2x[n-2].$$

Systems S_t and S_r are put into cascade. The resulting cascaded system is called S_z.

(c) (2 pt) Give the impulse response of system S_z.

We consider an input signal:

$$x_1[n] = 3\delta[n] - \delta[n-2].$$

(d) (1 pt) Compute the output $y_1[n]$ of system S_z for $x_1[n]$.
(e) (1 pt) Compute the output $y_2[n]$ of system S_z for input signal $x_2[n] = x_1[n] - 3x_1[n-1]$.

Question 2 (5 points)

We consider a system S_1, characterized by the following input-output relation:

$$S_1: \qquad y[n] = \frac{1}{4}y[n-1] + 2x[n] - x[n-1].$$

For $n < 0$, the system is in rest.

(a) (2 pt) Give the impulse response $h_1[n]$ of system S_1 for all values $n \in [-\infty, \infty]$.

(b) (1 pt) Explain whether S_1 is a stable IIR filter. Motivate your answer in at most 75 words.

We give an input signal x_3 to the system, which is specified as follows:

n	≤ -1	0	1	2	3	4	≥ 5
$x_3[n]$	0	2	4	6	0	0	0

(c) (2 pt) Compute the output of system S_1 for input signal $x_3[n]$.

Question 3 (7 points)

We consider a continuous-time periodic signal:

$$x_4(t) = 3\cos\left(100\pi t + \frac{\pi}{2}\right).$$

(a) **(1 pt)** Make a clear plot of signal $x_4(t)$ over two periods, starting at $t = 0$.

Signal $x_4(t)$ is sampled at 400 Hz, resulting in a discrete-time signal $x_4[n]$.

(b) **(1 pt)** Give an expression for signal $x_4[n]$ and make a plot of the signal $x_4[n]$ over two periods, starting at $n = 0$.

We now consider two more continuous-time periodic signals:

$$x_5(t) = 6\cos\left(500\pi t - \frac{\pi}{4}\right).$$

$$x_6(t) = 5 + x_4(t) + x_5(t).$$

(c) **(1 pt)** What is the Shannon-Nyquist sampling rate of $x_5(t)$? Motivate your answer in at most 75 words.

(d) **(1 pt)** What is the fundamental frequency of $x_6(t)$? Motivate your answer in at most 75 words.

(e) **(2 pt)** Express $x_6(t)$ as a sum of complex exponents.

(f) **(1 pt)** Make a clear plot of the complex spectrum *X*(*f*) of $x_6(t)$. Label each component in the complex spectrum with the appropriate complex amplitude.

Question 4 (9 points)

In broadcast content, spoken text that is deemed unsuitable or sensitive for the public ear is often censored by a beep. Consider we have a sampled audio recording $s[n]$, featuring speech uttered by multiple speakers. Some parts of the audio recording are censored by a censoring beep of 1000 Hz. Human speech typically has fundamental frequencies between 85 and 400 Hz. The audio recording is sampled at $F_s = 4000$ Hz.

We are interested in detecting where in time the censored content occurs. To this end, a framework is implemented applying the following steps (illustrated in Figure 1):

- Employ a short-time analysis window to consider consecutive smaller parts (analysis frames) of the recording. An analysis frame is denoted by $x[n]$.
- For each frame, apply an LTI filter S_{beep} retaining frequencies associated with a censor beep, and a filter S_{speech} retaining frequencies associated with speech. As a consequence, we obtained filtered outcomes $y_{beep}[n]$ and $y_{speech}[n]$.
- Take the DFT of $y_{beep}[n]$ and $y_{speech}[n]$, and compute the sum of squared DFT coefficients for $Y_{beep}[k]$ and $Y_{speech}[k]$.
- If $\Sigma(Y_{beep}[k])^2 \gg \Sigma(Y_{speech}[k])^2$, the frame is marked as having censored content.

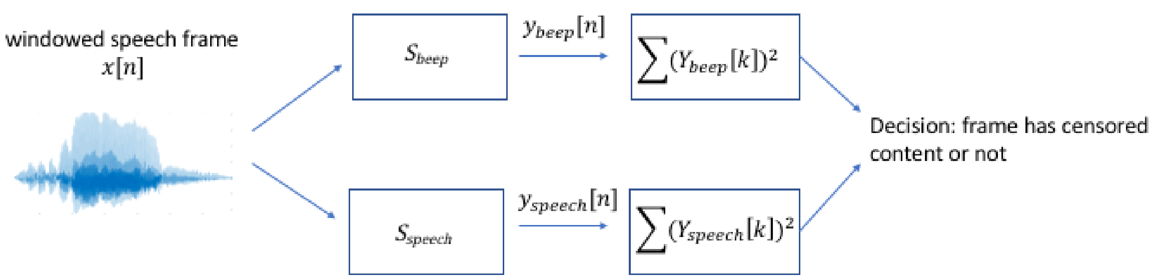

Figure 1. Censored content assessment framework.

(a) **(2 pt)** Consider the characteristics of the DTFT magnitude spectrum $|S(e^{j\hat{\omega}})|$ over the full audio recording $s[n]$. Make a sketch of what $|S(e^{j\hat{\omega}})|$ could look like, clearly indicating where on the frequency axis you would expect to see speech and beep occurrences. Explain why the DTFT would look like that in your view.

(b) **(1 pt)** Consider S_{beep} and S_{speech}, the filters that would be capable of retaining beeps and speech, respectively. If you could freely design suitable frequency responses for S_{beep} and S_{speech}, what would these look like? Make a clear sketch and explain your drawing.

We now consider three possible LTI filters:

- filter A, with $h_A[n] = \delta[n] + \delta[n-2]$.
- filter B, with $h_B[n] = \delta[n] + \delta[n-4]$.
- filter C, with $h_C[n] = \delta[n-1] + \delta[n-2] + \delta[n-3]$.

(c) **(1 pt)** Compute the frequency response $H_A(e^{j\hat{\omega}})$ for filter A and sketch the corresponding magnitude spectrum.

(d) **(1 pt)** Compute the frequency response $H_B(e^{j\hat{\omega}})$ for filter B and sketch the corresponding magnitude spectrum.

(e) **(1 pt)** Compute the frequency response $H_C(e^{j\hat{\omega}})$ for filter C and sketch the corresponding magnitude spectrum.

(f) **(1 pt)** Make a clear plot of the phase $\arg\left(H_c\left(e^{j\hat{\omega}}\right)\right)$

None of the filters A, B, C would be optimal for S_{beep} or S_{speech}. However, by cascading two of these filters, a reasonable filter for S_{speech} can be obtained.

(g) **(2 pt)** Which two filters out of {A, B, C} would you put in cascade for S_{speech}? Explain your answer and make a sketch of the magnitude spectrum of the cascaded filter.

Question 5 (9 points)

We consider a signal $x_7[n]$:

$$x_7[n] = 2\delta[n] - \delta[n-1] + 2\delta[n-4].$$

We now wish to compute the DFT $X_7[k]$ of signal $x_7[n]$. For this, we need to choose the length of the DFT.

(a) (1 pt) Choosing a larger DFT (for example, a 1024-point DFT instead of a 4-point DFT) will give a closer approximation to the DTFT. Explain why this is the case.

In part (b) and (c), we choose for $X_7[k]$ to be an 8-point DFT of signal $x_7[n]$. An incomplete specification of $X_7[k]$ is given below:

k	0	1	2	3	4	5	6	7
$X_7[k]$	?	$-0.7071 + 0.7071j$	?	?	15	$0.7071 - 0.7071j$	?	?

(b) (1 pt) Calculate $X_7[0]$.

(c) (1 pt) Calculate $X_7[2]$.

(d) (1 pt) Give the full specification of $X_7[k]$ using the properties of the DFT of a real-valued signal. Explain how you determined the missing values (?).

Signal $x_7[n]$ is given as input to a system S_F, yielding an output signal $y_7[n]$. The system has the following impulse response $h_f[n]$:

n	0	1	2	3	4	5	6	7
$h_f[n]$	1.5	-0.5	1.5	-0.5	1.5	-0.5	1.5	-0.5

The corresponding frequency response $H_F[k]$ is:

k	0	1	2	3	4	5	6	7
$H_F[k]$	4	0	0	0	8	0	0	0

The output signal $y_7[n]$ can be computed in multiple ways. One way to do is, is by first computing $Y_7[k]$, making use of $X_7[k]$ and $H_F[k]$.

(e) (1 pt) Compute $Y_7[k]$, making use of $X_7[k]$ and $H_F[k]$.

(f) **(2 pt)** Compute the output signal $y_7[n]$ by taking the inverse DFT of $Y_7[k]$. (Hint: for the inverse transformation, use the formula $x[n] = \frac{1}{N}\sum_{K=0}^{N-1} X[K]e^{j\frac{2\pi}{N}Kn}$)

While $y_7[n]$ can be computed by taking the inverse DFT of $Y_7[k]$, it can alternatively be computed through a linear convolution of $x_7[n]$ and $h_f[n]$.

(g) (1 pt) Compute $y_7[n]$ through linear convolution.

(h) (1 pt) Is the $y_7[n]$ computed by direct convolution the same to the $y_7[n]$ obtained via the inverse DFT in (f)? Explain why the results are/are not the same in around 75 words.