Signal Processing TI2716-A

Answers Partial exam 2
November 3, 2015, 13.30-15.30 h

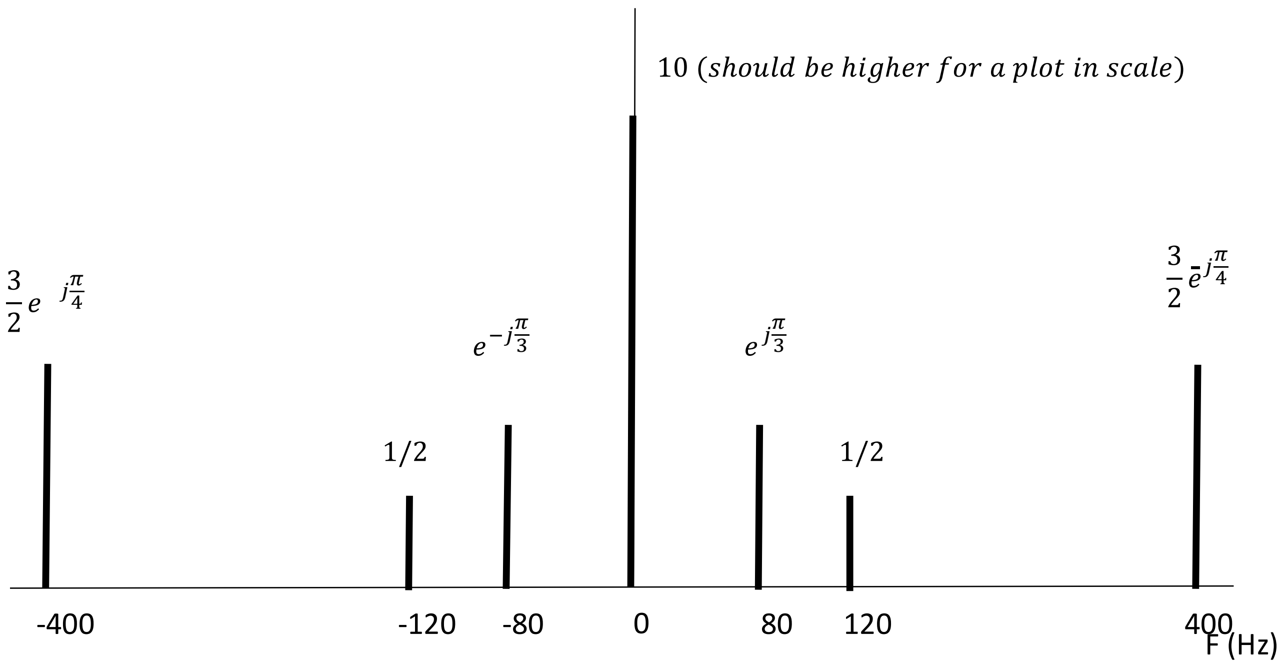
Question 1 (10 points)

(a) (1 pt) The sinusoidal components are of the form $Acos(2\pi f t + \phi)$. The fundamental frequency f_0 is the greatest common divisor for the different frequencies f. Therefore, $f_0 = 40$ Hz.

(b) (2 pt)

$$x_1(t) = 10 + e^{j\frac{\pi}{3}}e^{j160\pi t} + e^{-j\frac{\pi}{3}}e^{-j160\pi t} + \frac{1}{2}e^{j240\pi t} + \frac{1}{2}e^{-j240\pi t} + \frac{3}{2}e^{-j\frac{\pi}{4}}e^{j800\pi t} + \frac{3}{2}e^{j\frac{\pi}{4}}e^{-j800\pi t}$$

(c) (1 pt)



(d) (1 pt) $x_1[n] = 10 + 2\cos\left(\frac{160}{1200}\pi n + \frac{\pi}{3}\right) + \cos\left(\frac{240}{1200}\pi n\right) + 3\cos\left(\frac{800}{1200}\pi n - \frac{\pi}{4}\right)$

(e) (1 pt) $h[n] = \begin{cases} (0.3)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$

(f) (2 pt) $H\left(e^{j\hat{\omega}}\right) = \frac{1}{1-0.3e^{-j\hat{\omega}}}$

(e) (1 pt)

10: amplified

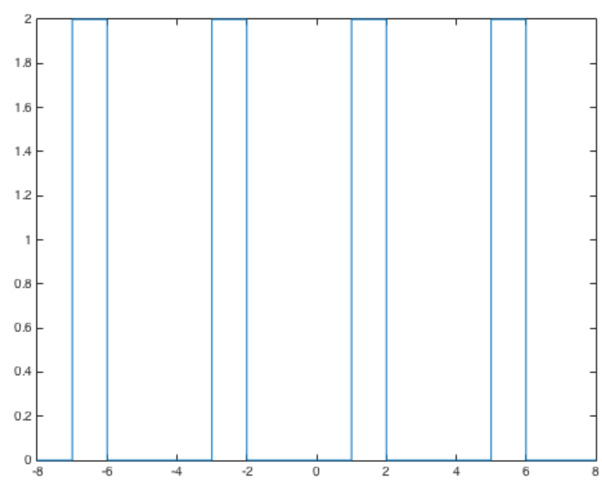
$2\cos\left(\frac{160}{1200}\pi n + \frac{\pi}{3}\right)$: amplified

$\cos\left(\frac{240}{1200}\pi n\right)$: amplified (about the same)

$3\cos\left(\frac{800}{1200}\pi n - \frac{\pi}{4}\right)$: attenuated

Question 2 (10 points)

(a) (1 pt)



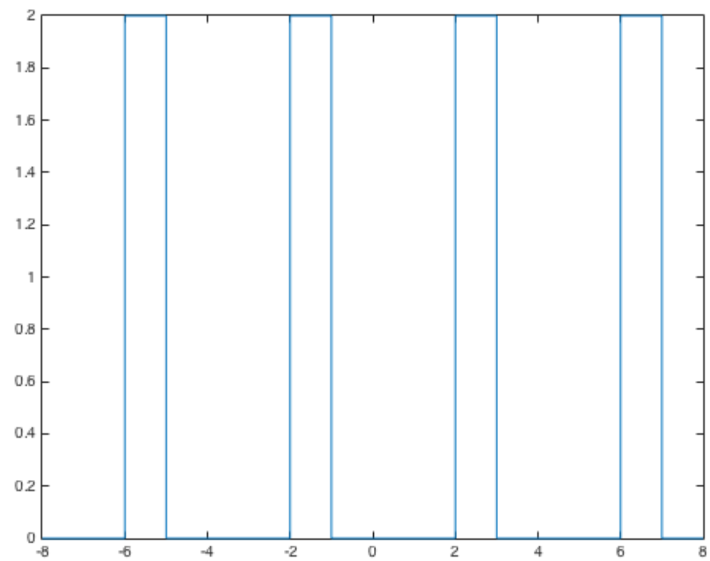
(b) (2 pt)

$$a_0 = \frac{1}{4}\int_0^4 x_2(t)dt = \frac{1}{4}\int_1^2 2dt = \frac{1}{2}$$

$$a_1 = \frac{1}{4}\int_0^4 x_2(t)e^{-j2\pi 1 f_0 t}dt \quad \text{with } f_0 = \frac{1}{T_0} = \frac{1}{4}$$

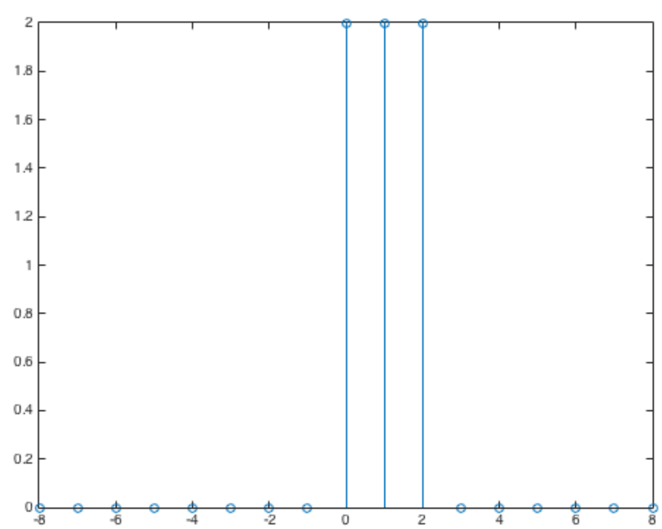
$$a_1 = \frac{1}{4}\int_1^2 2e^{-j\frac{\pi}{2}t}dt = \frac{1}{4}\frac{1}{-j\frac{\pi}{2}}\left[e^{-j\frac{\pi}{2}t}\right]_1^2 = \frac{2}{-2j\pi}\left(e^{-j\pi} - e^{-\frac{j\pi}{2}}\right) = -\frac{1}{\pi}(j+1)$$

(c) (1 pt)

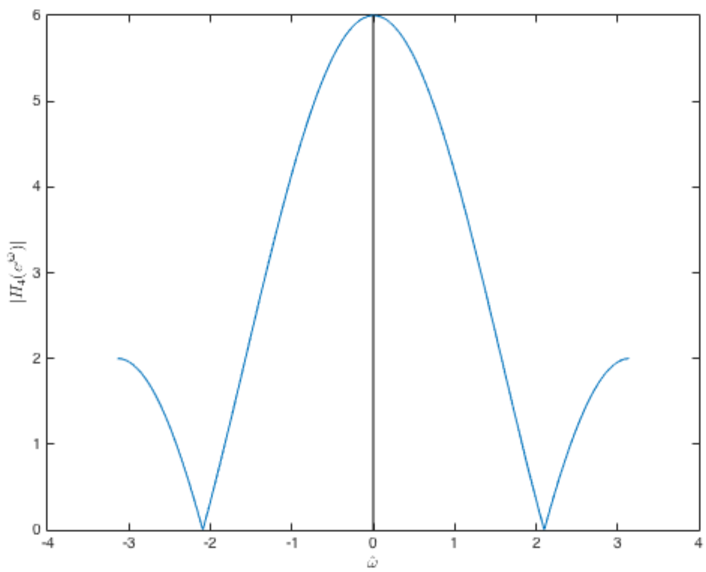


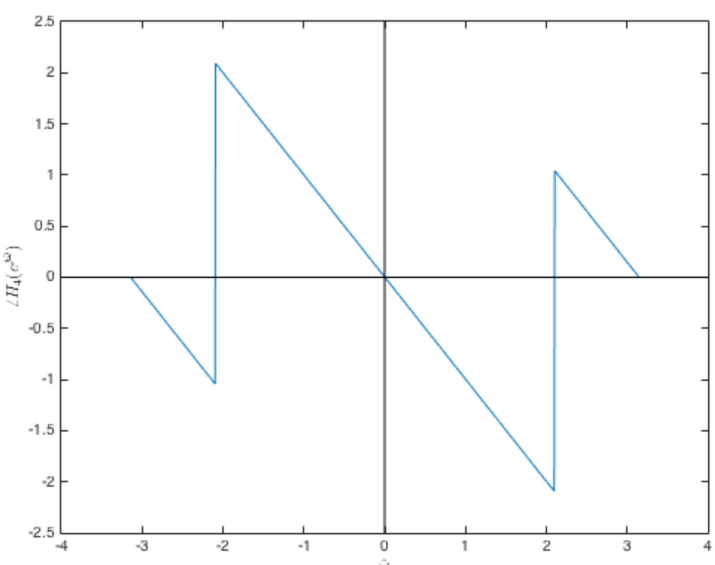
(d) (1 pt) The amplitude will remain the same. Only a phase shift will occur due to the time delay.

(e) (1 pt)



(f) (2 pt) $X_4\left(e^{j\widehat{\omega}}\right) = 2\left(1+e^{-j\widehat{\omega}} + e^{-2j\widehat{\omega}}\right) = 2e^{-j\widehat{\omega}}\left(e^{j\widehat{\omega}} + 1 + e^{-j\widehat{\omega}}\right) = 2e^{-j\widehat{\omega}}(1 + 2\cos(\widehat{\omega}))$



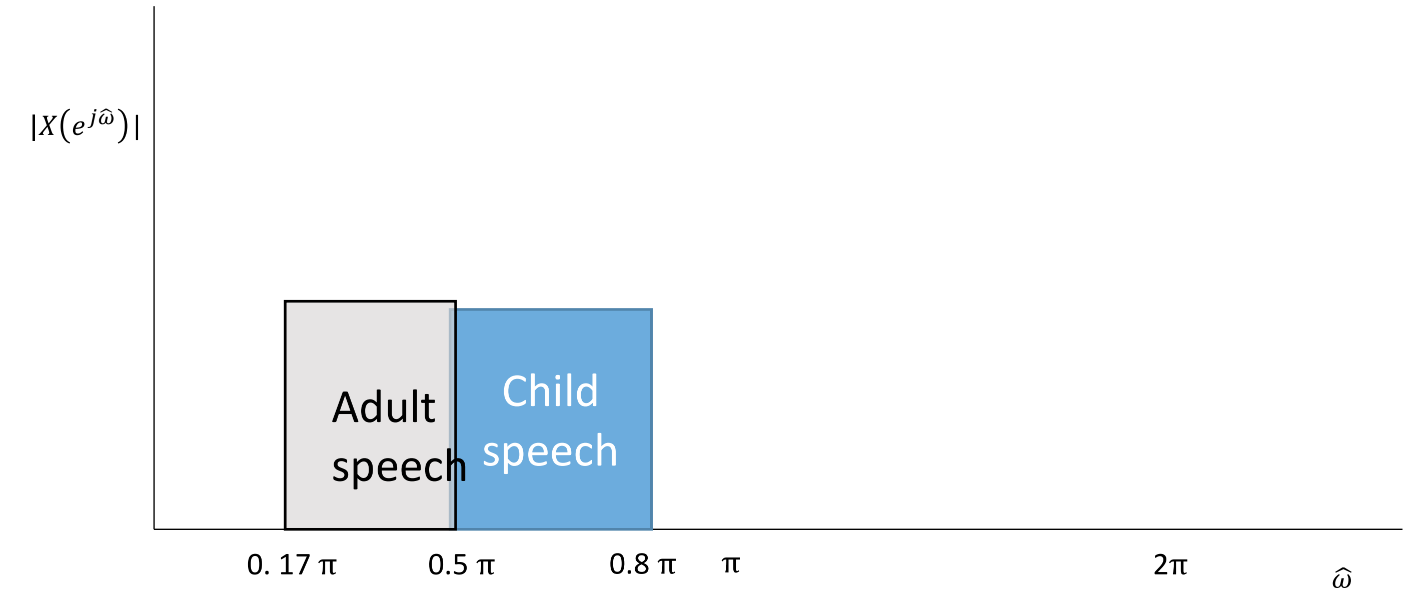


(g) (2 pt) $X_5[0] = \sum_{n=0}^{7} x[n] = 6.$

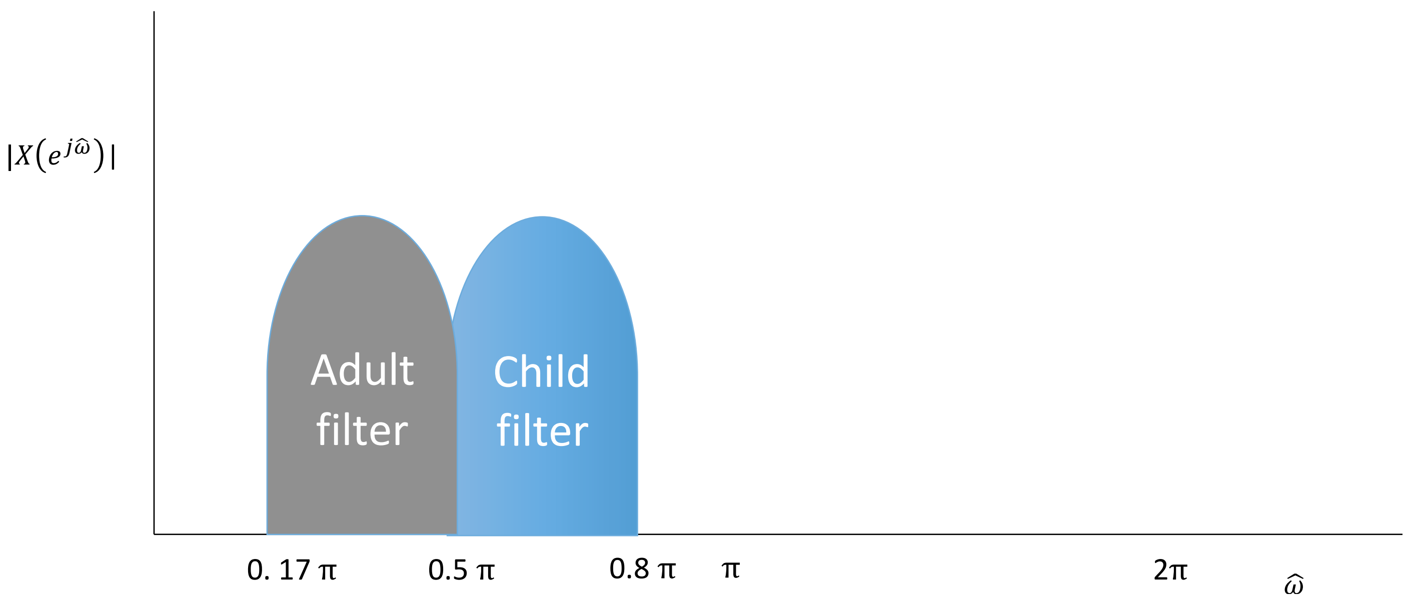
$$X_5[2] = \sum_{n=0}^{7} x[n] e^{-j\frac{\pi}{2}n} = 2 + 2e^{-j\frac{\pi}{2}} + 2e^{-j\pi} = 2 - 2j - 2 = -2j$$

Question 3 (10 points)

(a) (1 pt)



(b) (1 pt)



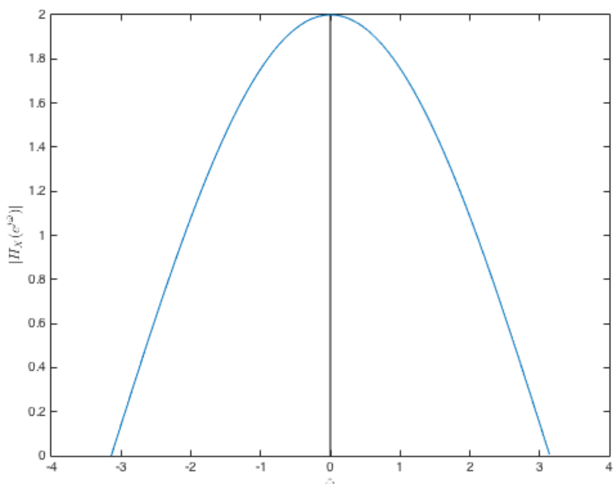
(c) (1 pt)

n	0	1	2	3	4	≥5
$h_X[n]$	1	1	0	0		0

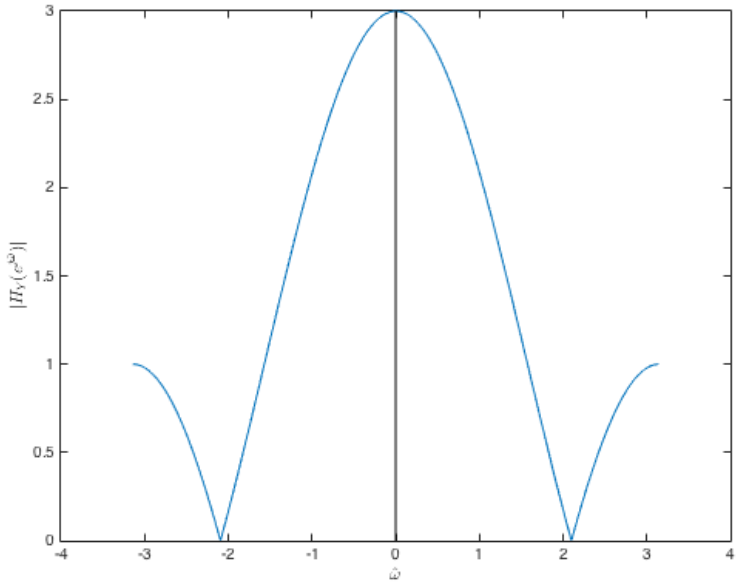
$h_Y[n]$	0	1	1	1	0	0
$h_Y[0]h_X[n]$	0	1	1	0	0	0
$h_Y[1]h_X[n-1]$	0	0	1	1	0	0
$h_Y[2]h_X[n-2]$	0	0	0	1	1	0
$h_Y[n] * h_X[n]$	0	1	2	2	1	0

$$h_Z[n] = \delta[n-1] + 2\delta[n-2] + 2\delta[n-3] + \delta[n-4]$$

(d) (2 pt) $H_X(e^{j\hat{\omega}}) = 1 + e^{-j\hat{\omega}} = e^{-j\frac{\hat{\omega}}{2}}\left(e^{j\frac{\hat{\omega}}{2}} + e^{-j\frac{\hat{\omega}}{2}}\right) = 2e^{-j\frac{\hat{\omega}}{2}}\cos\left(\frac{\hat{\omega}}{2}\right)$

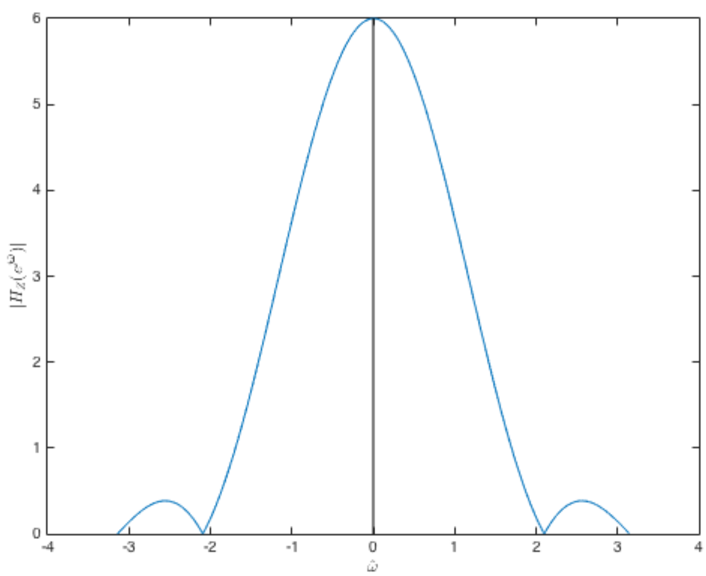


(e) (2 pt) $H_Y(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + e^{-2j\hat{\omega}} + e^{-3j\hat{\omega}} = e^{-2j\hat{\omega}}\left(1 + e^{j\hat{\omega}} + e^{-j\hat{\omega}}\right) = e^{-2j\hat{\omega}}(1 + 2\cos(\hat{\omega}))$



(f) (2 pt) $H_Z(e^{j\hat{\omega}}) = e^{-j\hat{\omega}} + 2e^{-2j\hat{\omega}} + 2e^{-3j\hat{\omega}} + e^{-4j\hat{\omega}}$

For the corresponding magnitude, one can e.g. multiply the magnitude spectra for filters X and Y.



(g) (1 pt) We wish to retain the lower-frequency adult speech, while removing the higher-frequency child speech. Following the magnitude plots, this is best achieved with filter Z.