Second Partial Exam Signal Processing TI2716-A

November 5$^{\text{th}}$, 2014
14:00 - 17:00 h

- This exam has 5 questions, for which a total of 40 points can be obtained.
- The result of this exam has a weight of 70% in the overall result.
- The allotted time for this exam is 3 hours.
- Use of the Equation Sheet TI2716-A is permitted.
- Please answer each question on a new sheet of paper.
- Questions may be answered in Dutch or English.

Question 1 (8 points total)

We consider the following time-discrete periodic signal:

$$x[n] = 5 + 3\cos\left(\frac{\pi}{2}n + \frac{\pi}{3}\right) + \cos\left(\pi n - \frac{\pi}{2}\right) + 4\cos\left(\frac{\pi}{4}n\right).$$

(a) (2 p.) Rewrite $x[n]$ in terms of a sum of complex exponentials.

If we consider the complex spectrum $X(e^{j\hat{\omega}})$ belonging to $x[n]$, we will obtain several non-zero values in the complex spectrum.

(b) (1 p.) Considering the complex spectrum $X(e^{j\hat{\omega}})$ for $-\pi \leq \hat{\omega} \leq \pi$, at which values of $\hat{\omega}$ do we obtain non-zero values?

(c) (2 p.) Sketch the complex spectrum $X(e^{j\hat{\omega}})$ for $-\pi \leq \hat{\omega} \leq \pi$, and clearly indicate the value of the complex amplitudes. In other words, sketch the result of applying the DTFT to $x[n]$.

We wish to compute an N-point DFT of $x[n]$ in Matlab, employing the Fast Fourier Transform (FFT). The resulting DFT of $x[n]$ is indicated by $X[k]$. The FFT can only be used if N is a power of 2.

(d) (1 p.) What is the smallest value of N for which we can apply the FFT, such that we obtain exactly the same number of non-zero values in $X[k]$ (for $0 \leq k \leq N-1$) as you indicated in question (c)?

(e) (2 p.) Assume we choose $N = 16$ and apply the FFT to $x[n]$. Plot the magnitude of the resulting spectrum $X[k]$ as a function of k. Use clear labels and indication of values for both axes.

Question 2 (10 points total)

An input-output system $\mathcal{S}_1$ is specified by the following input-output relation:

$$y[n] = 4x[n] - 3x[n-1] + 6x[n-2].$$

Next to that, an input-output system $\mathcal{S}_2$ is specified by the following input-output relation:

$$y[n] = 2x[n-1] + x[n-2].$$

System $\mathcal{S}_1$ and $\mathcal{S}_2$ are put in cascade as indicated in Figure 1, yielding system $\mathcal{S}_3$.

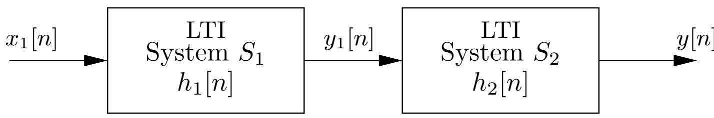

Figure 1: Cascaded system $\mathcal{S}_3$, composed of the LTI systems $\mathcal{S}_1$ and $\mathcal{S}_2$.

(a) (2 p.) Determine the impulse responses $h_1[n]$ of system $\mathcal{S}_1$, $h_2[n]$ of system $\mathcal{S}_2$, and compute the impulse response $h_3[n]$ of system $\mathcal{S}_3$ by direct (linear) convolution.

(b) (1 p.) Compute the transfer function $H_1(e^{j\hat{\omega}})$.

(c) (2 p.) Compute the transfer function $H_2(e^{j\hat{\omega}})$ and make a sketch of $|H_2(e^{j\hat{\omega}})|$.

(d) (2 p.) Show that $H_3(e^{j\hat{\omega}}) = H_1(e^{j\hat{\omega}})H_2(e^{j\hat{\omega}})$.

An unknown signal $x[n]$ is given as input to system S_3. The observed output signal $y[n]$ is:

$$y[n] = 8\delta[n-3] - 2\delta[n-4] + 9\delta[n-5] + 6\delta[n-6].$$

(e) (1 p.) Compute DTFT $Y(e^{j\hat{\omega}})$ of $y[n]$.

(f) (2 p.) Make use of the result in question (d) and (e) to compute the unknown input signal $x[n]$ and its DTFT $X(e^{j\hat{\omega}})$.

Question 3 (6 points total)

We consider the following LTI system (IIR filter), which initially is at rest:

$$y[n] = -\frac{1}{4}y[n-1] + 2x[n].$$

(a) (1 p.) Determine the impulse response $h[n]$ of this system.

(b) (2 p.) Compute the frequency response $H(e^{j\hat{\omega}})$ of the IIR filter.

On the handout sheet, you can find incomplete plots of the magnitude and phase of $H(e^{j\hat{\omega}})$.

(c) (1 p.) Complete the magnitude and phase plots of $H(e^{j\hat{\omega}})$ for $-\pi \leq \hat{\omega} \leq \pi$.

The following signal $x[n]$ is given as input to the system:

$$x[n] = \cos\left(\frac{\pi}{5}n - \frac{\pi}{5}\right) + 4\cos\left(\frac{3\pi}{4}n\right),$$

yielding an output signal $y[n]$ of the following form:

$$y[n] = A_0 \cos\left(\frac{\pi}{5}n + \phi_0\right) + A_1 \cos\left(\frac{3\pi}{4}n + \phi_1\right).$$

(d) (2 p.) Use the plot of the magnitude of $H(e^{j\hat{\omega}})$ on the handout sheet to estimate the value of A_0 and A_1 (no exact computations required).

Question 4 (8 points total)

We consider the following signal $x[n]$:

$$x[n] = \delta[n] + \delta[n-1] + 2\delta[n-2] + 2\delta[n-6] + 4\delta[n-7].$$

(*Take good notice of the time-index of the impulse signals!*). We analyze the signal by considering its spectrum. To that end we use an $N = 8$-point DFT of $x[n]$, resulting in the spectrum $X[k]$.

(a) (1 p.) Calculate the DFT coefficient $X[0]$. What are the magnitude and phase of $X[0]$?

(b) (2 p.) Calculate the DFT coefficient $X[2]$. What are the magnitude and phase of $X[2]$?

(c) (1 p.) Write $X[2]$ in the form of a complex exponent, and plot its value in the complex plane.

(d) (1 p.) Which DFT coefficient $X[k]$ is equal to the the complex conjugated value of $X[2]$?

We wish to input $x[n]$ to a LTI system with input-output relation

$$y[n] = x[n] + x[n-1].$$

We compute the convolution of $x[n]$ with the impulse response $h[n]$ of the LTI system by multiplying their DFT spectra $X[k]$ and $H[k]$, i.e.,

$$Y[k] = H[k]X[k].$$

After applying the inverse DFT on $Y[k]$ we obtain the following convolution result:

$$y[n] = 5\delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + 2\delta[n-6] + 6\delta[n-7].$$

(e) (1 p.) The value $y[0] = 5$ is different from the result obtained via linear convolution. What is the result $y[0]$ obtained by linear convolution?

(f) (2 p.) Explain how the value $y[0] = 5$ is obtained as a consequence of computing the convolution in the frequency domain.

Question 5 (8 points total)

An analog recording of a speech signal turns out to be degraded by a loud low-frequency "hum" and a small amount of random noise "hiss". Your task is to improve the quality of the recording using a digital FIR filter. The following information is given:

- You sample the analog recording with a sampling rate of $F_s = 2000$ Hz.
- The frequency of the degrading loud "hum" consistently lies between 49 and 51 Hz.
- The small amount of random noise "hiss" appears at frequencies between 10 and 900 Hz.
- The speech signal contains mostly frequencies in the range of 350 and 700 Hz.

(a) (1 p.) What is the range of the digital frequency $\hat{\omega}$ of the "hum" ?

(b) (1 p.) Make a clear sketch of the DTFT amplitude spectrum of the sampled recording for $-\pi \leq \hat{\omega} \leq \pi$, indicating the spectral location of the speech signal, and the degrading "hum" and "hiss".

(c) (2 p.) Make a separate sketch in which you indicate the magnitude of the ideal transfer function $|H(e^{j\hat{\omega}})|$ for $-\pi \leq \hat{\omega} \leq \pi$, and motivate your answer.

Rather than using the ideal filter you selected in question (c), you decide to work with a set of simple filters. In particular you have available the following filters:

- Filter S_1 with impulse response $h_1[n] = 2\delta[n] + 2\delta[n-1]$;
- Filter S_2 with impulse response $h_2[n] = 2\delta[n] - 2\delta[n-1]$;
- Filter S_3 which is the result of putting the filters S_1 and S_2 in cascade.

(d) (1 p.) Compute the frequency response $H_1(e^{j\hat{\omega}})$ of filter S_1. Sketch a plot of the corresponding magnitude spectrum for $-\pi \leq \hat{\omega} \leq \pi$.

(e) (1 p.) Show that the magnitude response $|H_2(e^{j\hat{\omega}})|$ of filter S_2 equals $|4\sin(\frac{\hat{\omega}}{2})|$. Sketch a plot of the magnitude spectrum for $-\pi \leq \hat{\omega} \leq \pi$.

(f) (1 p.) Sketch a plot of the magnitude spectrum $|H_3(e^{j\hat{\omega}})|$ of filter S_3 for $-\pi \leq \hat{\omega} \leq \pi$.

(g) (1 p.) Which of the filters S_1, S_2, S_3 will give the best result in suppressing both the degrading "hum" and "hiss"? Explain your answer.

Final exam TI2716-A
November 5, 2014
Handout sheet for assignment 3 (c)

Name:

Study number:

Complete the figures below according to the instructions in the exam. Please hand in this sheet together with your other solutions.