Exam Signal Processing TI2710-A

November 7th, 2012

Question 1 - Linear time-invariant (LTI) systems (8 points total)

A discrete-time system is described by the following input-output relation:

$$y[n] = x[n+1]x[n-1] + 2x[n].$$

(a) (2 p.) Show whether or not this system is linear.

(b) (2 p.) Show whether or not this system is time-invariant.

Given are two LTI systems S_1 and S_2 with the following impulse responses:

$$S_1: \quad h_1[n] = \delta[n] - \delta[n-1],$$

and

$$S_2: \quad h_2[n] = 2\delta[n] + 2\delta[n-1] + 2\delta[n-2].$$

These two LTI systems are cascaded as illustrated in Figure 1.

(c) (2 p.) Show that the impulse response of the cascaded system S_3 equals

$$S_3: \quad h[n] = 2\delta[n] - 2\delta[n-3].$$

(d) (1 p.) Explain why the order in which the systems S_1 and S_2 are cascaded, does not influence the impulse response $h[n]$ of S_3.

(e) (1 p.) The input to the LTI system S_3 is given by $x[n] = 4\delta[n] + 2\delta[n-1]$. Compute the output $y[n]$ of S_3.

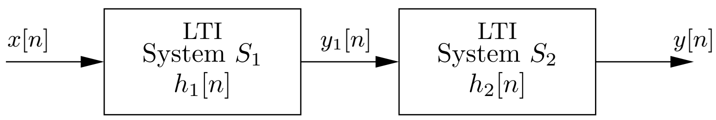

Figure 1: Cascaded system S_3, composed of the LTI systems S_1 and S_2.

Question 2 - Fourier series (7 points total)

Let a time-continuous periodic signal with period $T_0 = 3$ be described over one period by

$$z(t) = \begin{cases} 0 & 0 \leq t < \frac{3}{2} \\ -2 & \frac{3}{2} \leq t < 3. \end{cases}$$

The Fourier series analysis integral (continuous time, discrete frequency) is defined as

$$a_k = \frac{1}{T_0} \int_0^{T_0} z(t) e^{-j(2\pi/T_0)kt} dt.$$

(a) (1 p.) Compute the Fourier coefficient a_0 of the signal $z(t)$.

(b) (2 p.) Compute the Fourier coefficients a_{-1} and a_1 of the signal $z(t)$.

(c) (1 p.) Plot the complex spectrum of $z(t)$ using only the coefficients a_{-1}, a_0, and a_1. Clearly mark the scale of the axes of the complex spectrum.

From the Fourier coefficients a_{-1}, a_0, and a_1 we reconstruct the continuous-time signal $\tilde{z}(t)$.

(d) (2 p.) Give the expression for the reconstructed time-domain signal $\tilde{z}(t)$ based on the Fourier coefficients a_{-1}, a_0, and a_1.

(e) (1 p.) Sketch $z(t)$ and $\tilde{z}(t)$ in one figure. Explain the observed differences between $z(t)$ and $\tilde{z}(t)$.

Question 3 - Discrete-time Fourier transformations (7 points total)

A discrete-time signal $x[n]$ is given by

$$x[n] = \begin{cases} 3 & n = 0, \\ -3 & n = 2, \\ 0 & \text{all other values of } n. \end{cases}$$

(a) (2 p.) Compute the DTFT of this signal and show that it equals

$$X(e^{j\hat{\omega}}) = 6e^{-j\hat{\omega}+j\frac{\pi}{2}} \sin(\hat{\omega}).$$

(b) (1 p.) Give the expression for the magnitude of $X(e^{j\hat{\omega}})$, and make a plot of $|X(e^{j\hat{\omega}})|$.

(c) (1 p.) Give the expression for the phase of $X(e^{j\hat{\omega}})$, and make a plot of $\arg(X(e^{j\hat{\omega}}))$.

The signal $x[n]$ is input to an LTI system of which the impulse response is given by: $h[n] = \delta[n-1]$. The output of the LTI system is $y[n]$.

(d) (2 p.) Compute the DTFT $Y(e^{j\hat{\omega}})$ of the output $y[n]$.

(e) (1 p.) Plot the magnitude and phase spectrum of output $y[n]$.

Question 4 - Transfer functions (8 points total)

An LTI system is given by the input-output relation:

$$y[n] = x[n] + 3x[n-1] + x[n-2].$$

(a) (1 p.) Determine the impulse response $h[n]$, and clearly explain how you obtained your answer.

(b) (2 p.) Show that the transfer function of the LTI system equals:

$$H(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}}\left(\cos(\hat{\omega}) + \frac{3}{2}\right).$$

(c) (1 p.) Sketch the magnitude response of $H(e^{j\hat{\omega}})$ for $-3\pi \leq \hat{\omega} \leq 3\pi$. Clearly mark the axes and indicate zeroes and maxima in the sketch.

(d) (1 p.) Sketch the principal value of the phase response of $H(e^{j\hat{\omega}})$ for $-3\pi \leq \hat{\omega} \leq 3\pi$.

Consider the following input signal:

$$x[n] = 2 + \cos\left(\frac{\pi}{2}n + \frac{\pi}{4}\right).$$

(e) (1 p.) Plot the complex spectrum of $x[n]$. Clearly indicate the complex amplitudes $X(e^{j\hat{\omega}})$.

(f) (2 p.) Determine the output $y[n]$ of the filter $h[n]$ for the input signal $x[n]$ using the result obtained at (c) and (d).

Question 5 - Sampling and reconstruction (6 points total)

In a hospital a patient's electrocardiographic signal (ECG) is measured. The measurement yields the following time-continuous signal $x(t)$:

$$x(t) = \cos(2\pi 10t + \pi/2) + \cos(2\pi 50t + \pi/4) + \cos(2\pi 80t).$$

In order process (filter and analyze) the signal, we first convert the time-continuous signal $x(t)$ into a time-discrete signal $x[n]$.

(a) (1 p.) Choose a sampling frequency f_s. Motivate your choice (there is not a unique answer).

(b) (1 p.) For the sampling frequency selected under (a), give the expression for the resulting sampled time-discrete signal $x[n]$.

(c) (1 p.) Plot the complex spectrum of $x[n]$. Clearly indicate the complex amplitudes.

Two frequency components of the signal $x(t)$ are considered distortions, namely the signal components (cosines) with frequencies 10 Hz and 50 Hz. These distortions make it difficult to analyze the ECG signal of interest. For that reason we wish to develop a time-discrete filter that removes the signal components with frequencies 10 Hz and 50 Hz.

(d) (2 p.) Sketch the magnitude response $|H(e^{j\hat{\omega}})|$ of a discrete-time filter with input $x[n]$ and output $y[n]$, which will remove the signal components with frequencies 10 Hz and 50 Hz (there is not a unique answer).

From the samples of the filtered time-discrete signal $y[n]$ we reconstruct a time-continuous signal $y(t)$ by interpolating the samples.

(e) (1 p.) Choose an interpolation method. Then plot four periods of the time-discrete signal $y[n]$ and four periods of the resulting time-continuous interpolated signal $y(t)$.