Answers Exam Signal Processing TI2710-A

November 7th 2012

Question 1

(a) (2 p.) For a linear system, the superposition principle must hold. Let $x_1[n] \rightarrow y_1[n] = x_1[n+1]x_1[n-1] + 2x_1[n]$ and $x_2[n] \rightarrow y_2[n] = x_2[n+1]x_2[n-1] + 2x_2[n]$. Set now $x[n] = \alpha x_1[n] + \beta x_2[n] \rightarrow y[n] = (\alpha x_1[n+1] + \beta x_2[n+1])(\alpha x_1[n-1] + \beta x_2[n-1]) + 2(\alpha x_1[n] + \beta x_2[n]) \neq \alpha y_1[n] + \beta y_2[n] = \alpha(x_1[n+1]x_1[n-1] + 2x_1[n]) + \beta(x_2[n+1]x_2[n-1] + 2x_2[n])$. The system is thus not linear.

(b) (2 p.) The system is time-invariant: Delaying input $x[n]$ over n_0 samples, followed by filtering with the given filter, gives the same output as when $x[n]$ is first filtered followed by delaying the output. Hence: $x[n] \rightarrow x[n-n_0]$. $x[n-n_0] \rightarrow w[n] = x[n-n_0-1]x[n-n_0+1] + 2x[n-n_0]$, while first filtering gives $x[n] \rightarrow y[n] = x[n-1]x[n+1] + 2x[n]$, which after delaying over n_0 samples becomes $y[n-n_0] = x[n-n_0-1]x[n-n_0+1] + 2x[n-n_0]$. Clearly, $y[n-n_0]$ and $w[n]$ are equal.

(c) (2 p.) We can convolve the two impulse responses h_1 and h_2, which will lead to

$$h[n] = h_2[n] - h_2[n-1] = 2\delta[n] - 2\delta[n-3].$$

(d) (1 p.) The impulse response $h[n]$ of system S_3 is given by $h[n] = h_1 * h_2$. For convolution, the property of commutativity holds. This means that $h[n] = h_1 * h_2 = h_2 * h_1$ and thus, that the order doesn't matter.

(e) (1 p.)

n	0	1	2	3	4
$h[n]$	2	0	0	-2	0
$x[n]$	4	2	0	0	0
$x[0]h[n]$	8	0	0	-8	0
$x[1]h[n-1]$	0	4	0	0	-4
$y[n]$	8	4	0	-8	-4

$y[n]$ is thus given by $y[n] = 8\delta[n] + 4\delta[n-1] - 8\delta[n-3] - 4\delta[n-4]$

Question 2

(a) (1 p.) a_0 is the average of the signal in question. Therefore, $a_0 = -2 * (3 - 3/2)/3 = -1$.
or

$$a_0 = \frac{1}{3}\int_0^3 z(t)dt = \frac{-2}{3}\int_{3/2}^3 dt = \frac{-2}{3}[t]_{3/2}^3 = -1$$

(b) (2 p.)

$$a_1 = \frac{1}{3}\int_0^3 z(t)e^{-j(2\pi/3)t}dt = \frac{-2}{3}\int_{3/2}^3 e^{-j(2\pi/3)t}dt = \frac{-2}{3}\frac{3}{-j2\pi}\left(e^{-j(2\pi/3)3} - e^{-j(2\pi/3)3/2}\right)$$

$$= \frac{1}{j\pi}\left(1 - e^{-j\pi}\right) = \frac{2}{j\pi}$$

$a_{-1} = \overline{a_1} = \frac{2}{-j\pi}$

(c) (1 p.) See Fig. 1.

(d) (2 p.) $\tilde{z}(t) = \sum_{k=-1}^{k=1} a_k e^{(j2\pi/3)kt} = -1 + a_1 e^{(j2\pi/3)t} + a_{-1}e^{-(j2\pi/3)t} = -1 + \frac{2}{j\pi}e^{(j2\pi/3)t} - \frac{2}{j\pi}e^{-(j2\pi/3)t} = -1 + \frac{4}{\pi}\sin(2\pi t/3)$

(e) (1 p.) See Fig. 2.

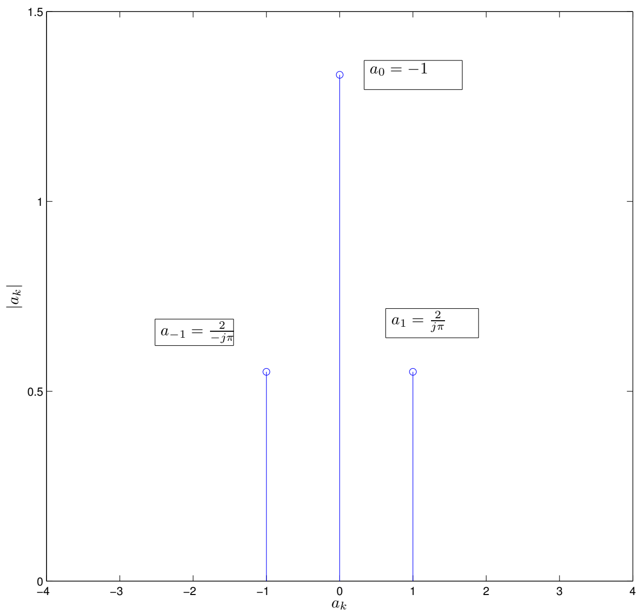

Figure 1: The complex spectrum of $z(t)$ based on a_{-1}, a_0 and a_1

Question 3

(a) (2 p.)

$$X(e^{j\hat{\omega}}) = \sum_n x[n]e^{-j\hat{\omega}n} = 3-3e^{-j2\hat{\omega}} = \frac{6j}{2j}e^{-j\hat{\omega}}(e^{j\hat{\omega}}-e^{-j\hat{\omega}}) = 6je^{-j\hat{\omega}}\sin(\hat{\omega}).$$

(b) (1 p.)

$$|X(e^{j\hat{\omega}})| = 6|\sin(\hat{\omega})|$$

For the plot, see See Fig. 3.

(c) (1 p.)

$$\angle X(e^{j\hat{\omega}}) = -\hat{\omega} + \frac{\pi}{2}$$

For the plot, see See Fig. 3.

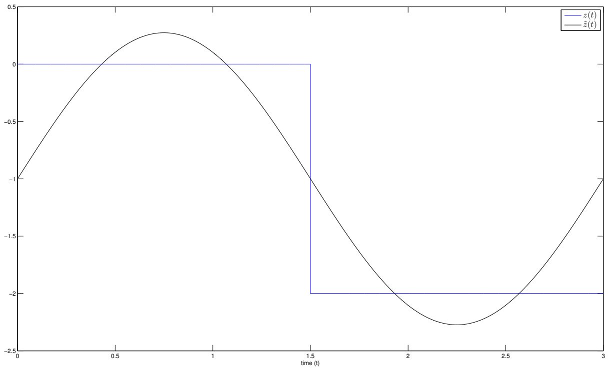

Figure 2: One period of $z(t)$ and $\tilde{z}(t)$.

(d) (2 p.) $H(\hat{\omega}) = e^{-j\hat{\omega}}$. This means that $Y(\hat{\omega}) = e^{-j\hat{\omega}} 6 j e^{-j\hat{\omega}} \sin(\hat{\omega}) = 6 j e^{-2j\hat{\omega}} \sin(\hat{\omega})$

(e) (1 p.) For the plot, see See Fig. 4.

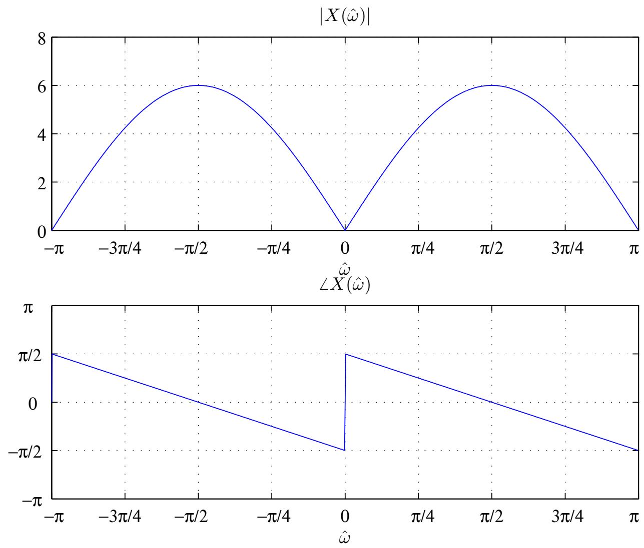

Figure 3: The magnitude and phase response of Question 3b and 3c.

Question 4

An LTI system is given by

$$y[n] = x[n] + 3x[n-1] + x[n-2].$$

(a) (1 p.) The impulse response is given by putting an impulse $x[n] = \delta[n]$ on the input, leading to $h[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$.

(b) (2 p.) $h[n] = \delta[n] + 3\delta[n-1] + \delta[n-2]$

$$H(e^{j\hat{\omega}}) = \sum_n h[n]e^{-j\hat{\omega}n} = 1 + 3e^{-j\hat{\omega}} + e^{-j2\hat{\omega}}$$

$$= \frac{2}{2}e^{-j\hat{\omega}}\left(e^{j\hat{\omega}} + e^{-j\hat{\omega}} + 3\right) = 2e^{-j\hat{\omega}}\left(\cos(\hat{\omega}) + \frac{3}{2}\right).$$

(c) (1 p.) See Fig. 5

(d) (1 p.) See Fig. 5

(e) (1 p.) $x[n] = 2 + \frac{1}{2}(e^{j(\frac{\pi n}{2})+\frac{\pi}{4}} + e^{-j(\frac{\pi n}{2})-\frac{\pi}{4}})$

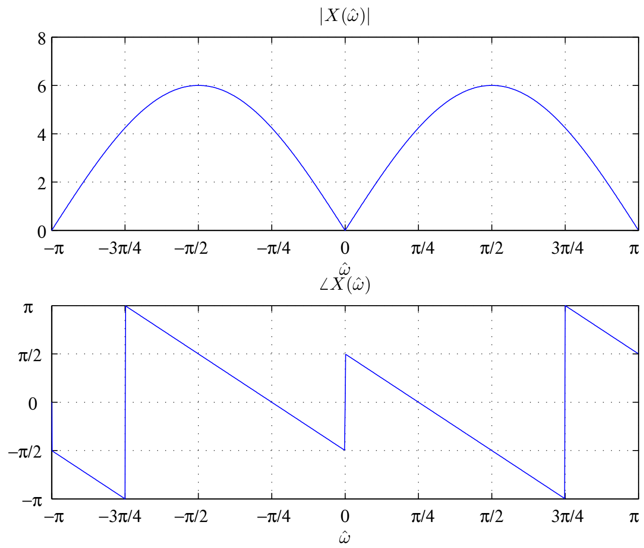

Figure 4: The magnitude and phase response of Question 3b and 3c.

(f) (2 p.)

$$x[n] = 2 + \frac{1}{2}e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + \frac{1}{2}e^{-j(\frac{\pi}{2}n+\frac{\pi}{4})}$$

The spectrum thus contains a delta pulse at $\hat{\omega} = 0$ of 2, a delta pulse at $\hat{\omega} = \pi/2$ of $\frac{1}{2}e^{j\frac{\pi}{4}}$ and a delta pulse at $\hat{\omega} = -\pi/2$ of $\frac{1}{2}e^{-j\frac{\pi}{4}}$ $H(\hat{\omega} = 0) = 5$ $H(\hat{\omega} = \frac{\pi}{2}) = 3e^{-j\frac{\pi}{2}}$

$$y[n] = 2 * 5 + 3e^{-j\frac{\pi}{2}}\frac{1}{2}e^{j(\frac{\pi}{2}n+\frac{\pi}{4})} + 3e^{j\frac{\pi}{2}}\frac{1}{2}e^{-j(\frac{\pi}{2}n+\frac{\pi}{4})}$$
$$= 10 + \frac{3}{2}e^{j(\frac{\pi}{2}n-\frac{\pi}{4})} + \frac{3}{2}e^{-j(\frac{\pi}{2}n-\frac{\pi}{4})} = 10 + 3\cos\left(\frac{\pi}{2}n - \frac{\pi}{4}\right)$$

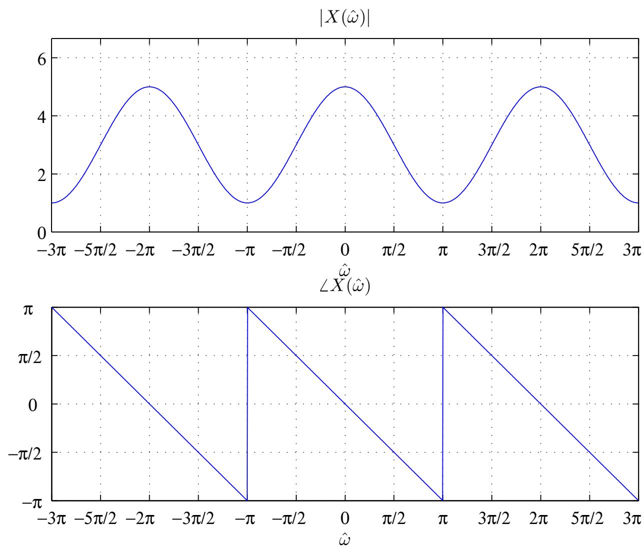

Figure 5: Magnitude and phase response Question 4c and 4d