Answers Second Partial Exam Signal Processing TI2716-A

November 5$^{\text{th}}$, 2014
14:00 - 17:00 h

Question 1 (8 points total)

(a) (2 p.) $x[n] = 5 + \frac{3}{2}e^{j\left(\frac{\pi}{2}n+\frac{\pi}{3}\right)} + \frac{3}{2}e^{-j\left(\frac{\pi}{2}n+\frac{\pi}{3}\right)} + \frac{1}{2}e^{j\left(\pi n-\frac{\pi}{2}\right)} + \frac{1}{2}e^{-j\left(\pi n-\frac{\pi}{2}\right)} + 2e^{j\frac{\pi}{4}n} + 2e^{-j\frac{\pi}{4}n}$
$= 5 + \frac{3}{2}e^{j\frac{\pi}{3}}e^{j\frac{\pi}{2}n} + \frac{3}{2}e^{-j\frac{\pi}{3}}e^{-j\frac{\pi}{2}n} + \frac{1}{2}e^{-j\frac{\pi}{2}}e^{j\pi n} + \frac{1}{2}e^{j\frac{\pi}{2}}e^{-j\pi n} + 2e^{j\frac{\pi}{4}n} + 2e^{-j\frac{\pi}{4}n}$

(b) (1 p.) Non-zero values in the spectrum will occur for $\hat{\omega} = 0, \frac{\pi}{2}, -\frac{\pi}{2}, \pi, -\pi, \frac{\pi}{4}, -\frac{\pi}{4}$.

(c) (2 p.) See Figure 1.

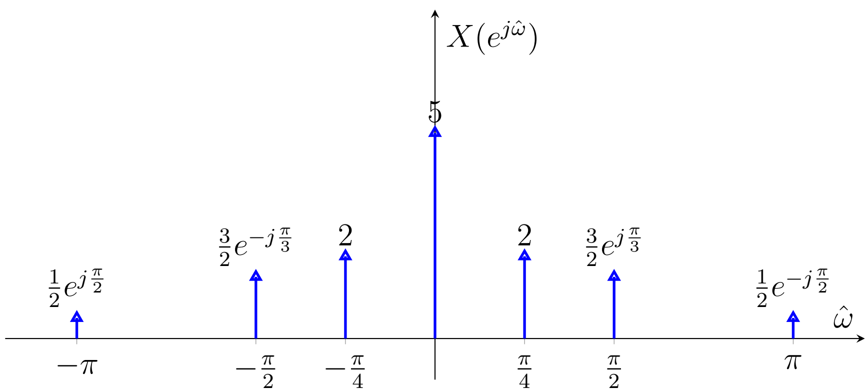

Figure 1: Plot corresponding to question 1(c).

(d) (1 p.) We need to choose N such that $\frac{\pi}{4}$, $\frac{\pi}{2}$ and π can be exactly represented. For this, we need to divide the $[0, 2*\pi]$ interval in at least 8 bins. So $N = 8$.

(e) (2 p.) See Figure 2 and revisit the hands-on exercises, assignment III.2 for related examples.

Question 2 (10 points total)

(a) (2 p.) $h_1[n] = 4\delta[n] - 3\delta[n-1] + 6\delta[n-2]$
$h_2[n] = 2\delta[n-1] + \delta[n-2]$
$h_3[n] = h_1[n] * h_2[n]$

$$\begin{bmatrix}4 & -3 & 6\end{bmatrix}\begin{bmatrix}0 & 2 & 1 & 0 & 0\\0 & 0 & 2 & 1 & 0\\0 & 0 & 0 & 2 & 1\end{bmatrix} = \begin{bmatrix}0 & 8 & -2 & 9 & 6\end{bmatrix}$$

So $h_3[n] = 8\delta[n-1] - 2\delta[n-2] + 9\delta[n-3] + 6\delta[n-4]$.

(b) (1 p.) $H_1(e^{j\hat{\omega}}) = 4 - 3e^{-j\hat{\omega}} + 6e^{-2j\hat{\omega}}$

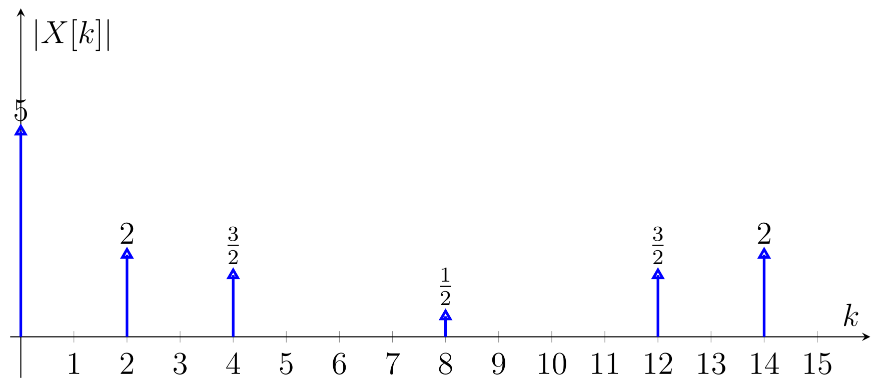

Figure 2: Plot corresponding to question 1(e).

(c) (2 p.) $H_2(e^{j\hat{\omega}}) = 2e^{-j\hat{\omega}} + e^{-2j\hat{\omega}}$
A plot of $|H_2(e^{j\hat{\omega}})|$ *is actually not trivial to make without any numerical aids. Therefore, this second part of the question was not taken into account for the results. For future resits of this exam, you will be expected to sketch magnitude plots for transfer functions of forms that were practised in the instruction sessions, such as functions of the form* $2e^{-j\hat{\omega}} + 2e^{-2j\hat{\omega}}$.

(d) (2 p.) From the answer to 2(a), we can deduce that $H_3(e^{j\hat{\omega}}) = 8e^{-j\hat{\omega}} - 2e^{-2j\hat{\omega}} + 9e^{-3j\hat{\omega}} + 6e^{-4j\hat{\omega}}$.
Verify that this equals the result of multiplying $H_1(e^{j\hat{\omega}})$ and $H_2(e^{j\hat{\omega}})$:
$H_1(e^{e^{j\hat{\omega}}}) \cdot H_2(e^{j\hat{\omega}}) = \left(4 - 3e^{-j\hat{\omega}} + 6e^{-2j\hat{\omega}}\right)\left(2e^{-j\hat{\omega}} + e^{-2j\hat{\omega}}\right)$
$= 8e^{-j\hat{\omega}} + 4e^{-2j\hat{\omega}} - 6e^{-2j\hat{\omega}} - 3e^{-3j\hat{\omega}} + 12e^{-3j\hat{\omega}} + 6e^{-4j\hat{\omega}}$
$= 8e^{-j\hat{\omega}} - 2e^{-2j\hat{\omega}} + 9e^{-3j\hat{\omega}} + 6e^{-4j\hat{\omega}} = H_3(e^{j\hat{\omega}})$.

(e) (1 p.) $Y(e^{j\hat{\omega}}) = 8e^{-3j\hat{\omega}} - 2e^{-4j\hat{\omega}} + 9e^{-5j\hat{\omega}} + 6e^{-6j\hat{\omega}}$

(f) (2 p.) $Y(e^{j\hat{\omega}}) = H_3(e^{j\hat{\omega}})e^{-2j\hat{\omega}}$. Thus, $X(e^{j\hat{\omega}}) = e^{-2j\hat{\omega}}$ and $x[n] = \delta[n-2]$.

Question 3 (6 points total)

(a) (1 p.) Due to the fact that we are dealing with a causal system we have

$$y[n] = 0 \quad \text{for } n < 0\,.$$

We calculate the coefficients of the impulse response by using $\delta[n]$ as input signal:

$$
\begin{aligned}
h[0] &= -\frac{1}{4}y[-1] + 2\delta[0] = \frac{1}{4}\cdot 0 + 2\cdot 1 = 2\\
h[1] &= -\frac{1}{4}y[0] + 2\delta[1] = -\frac{1}{4}\cdot 2 + 2\cdot 0 = 2\cdot\frac{1}{4}\\
h[2] &= -\frac{1}{4}y[1] + 2\delta[2] = -\frac{1}{4}\cdot(2\cdot -\frac{1}{4}) + 2\cdot 0 = 2\cdot(-\frac{1}{4})^2\\
h[3] &= -\frac{1}{4}y[2] + 2\delta[3] = -\frac{1}{4}\cdot(2\cdot(-\frac{1}{4})^2) + 2\cdot 0 = 2\cdot(-\frac{1}{4})^3\\
&\vdots\\
h[n] &= 2\cdot(-\frac{1}{4})^n\,, \quad \text{for } n = 0, 1, 2, \ldots .
\end{aligned}
$$

(b) (2 p.)

$$
\begin{aligned}
Y(e^{j\hat{\omega}}) &= -\frac{1}{4}e^{-j\hat{\omega}}Y(e^{j\hat{\omega}}) + 2X(e^{j\hat{\omega}})\\
Y(e^{j\hat{\omega}})(1 + \frac{1}{4}e^{-j\hat{\omega}}) &= 2X(e^{j\hat{\omega}})\,,
\end{aligned}
$$

resulting in

$$
H(e^{j\hat{\omega}}) = \frac{Y(e^{j\hat{\omega}})}{X(e^{j\hat{\omega}})} = \frac{2}{1 + \frac{1}{4}e^{-j\hat{\omega}}} \quad \text{for } -\pi \le \hat{\omega} \le \pi
$$

.

(c) (1 p.) See Figures 3 and 4.

The following signal $x[n]$ is given as input to the system:

$$
x[n] = \cos\left(\frac{\pi}{5}n - \frac{\pi}{5}\right) + 4\cos\left(\frac{3\pi}{4}n\right),
$$

yielding an output signal $y[n]$ of the following form:

$$
y[n] = A_0\cos\left(\frac{\pi}{5}n + \phi_0\right) + A_1\cos\left(\frac{3\pi}{4}n + \phi_1\right).
$$

(d) (2 p.) From the plot, at $\hat{\omega} = \frac{\pi}{5}$, the magnitude is approximately 1.7. At $\hat{\omega} = \frac{3\pi}{4}$, the magnitude is approximately 2.4.
Therefore, $A_0 \approx 1.7$ and $A_1 \approx 4 \cdot 2.4 = 9.6$ (note the multiplication factor of 4 in front of the corresponding cosine!).

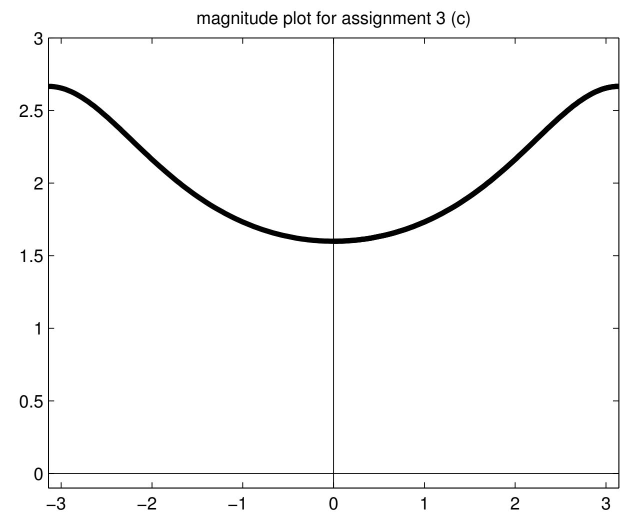

Figure 3: Magnitude plot for question 3(c).

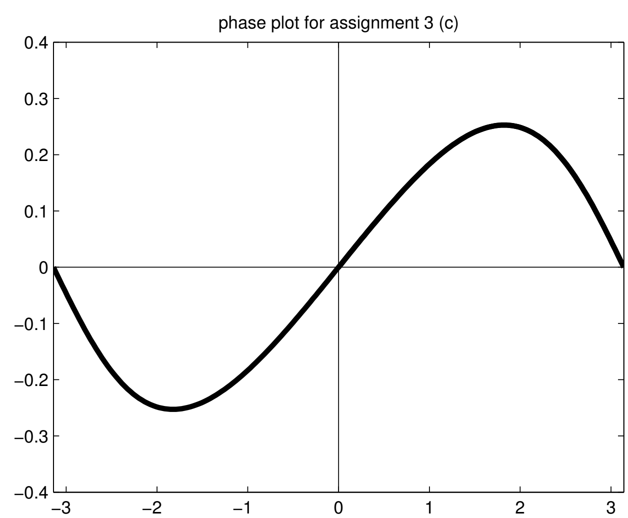

Figure 4: Phase plot for question 3(c).

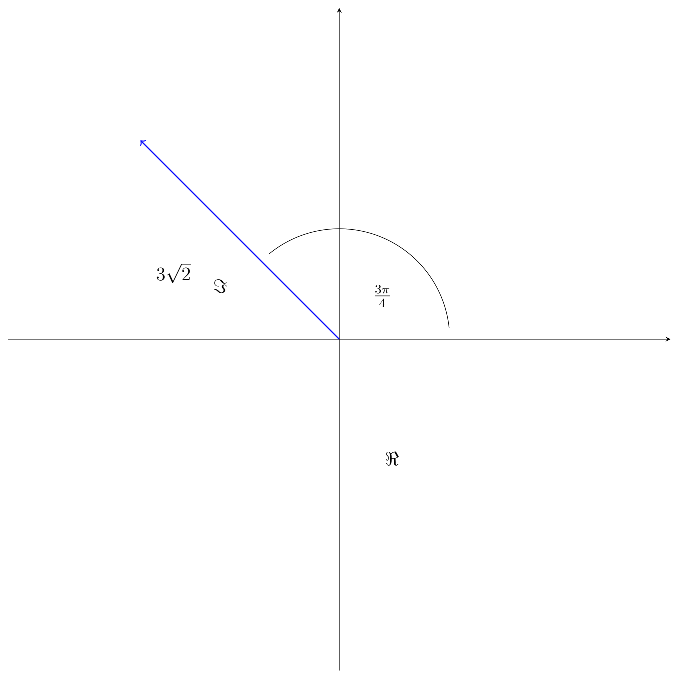

Figure 5: Plot corresponding to question 4(c).

Question 4 (8 points total)

(a) (1 p.) $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$
$X[0] = \sum_{n=0}^{7} x[n]e^{-j\frac{2\pi}{8}0\cdot n}$
$= 1 + 1 + 2 + 2 + 4 = 10$. $|X[0]| = 10$ and $\Phi[0] = 0$.

(b) (2 p.) $X[k] = \sum_{n=0}^{N-1} x[n]e^{-j\frac{2\pi}{N}kn}$
$X[2] = \sum_{n=0}^{7} x[n]e^{-j\frac{2\pi}{8}2\cdot n}$
$= 1 + e^{-j\frac{\pi}{2}} + 2e^{-j\pi} + 2e^{-j3\pi} + 4e^{-j\frac{7\pi}{2}} = 1 + -j + -2 + -2 + 4j = -3 + 3j$. $|X[2]| = \sqrt{(}18) = 3\sqrt{(}2)$ and $\Phi[2] = \frac{3\pi}{4}$.

(c) (1 p.) $X[2] = 3\sqrt{(}2)e^{j\frac{3\pi}{4}}$. For the plot, see Figure 5.

(d) (1 p.) The complex conjugated value of $X[2]$ can be found at $X[6]$.

(e) (1 p.) $y[0] = x[0]h[0] + x[-1]h[1] = 1$.

(f) (2 p.) When computing the convolution in the frequency domain, circular convolution is applied. Therefore, instead of taking $x[-1]$, we will take $x[N-1]$ for the second term. Thus, $y[0] = x[0]h[0] + x[7]h[1] = 1 + 4 = 5$.

Question 5 (8 points total)

(a) (1 p.) The "hum" lies between 49 and 51 Hz. In digital frequencies, this means that $\frac{2*pi*49}{2000} \leq \hat{\omega} \leq \frac{2*pi*51}{2000}$, so $0.049\pi \leq \hat{\omega} \leq 0.051\pi$.

(b) (1 p.) See Figure 6.

(c) (2 p.) A possible answer can be found in Figure 7. This would be a very straightforward 'ideal' transfer function, since everything outside of the frequency range of the speech signal would be removed from the signal. (In practice, with this transfer function, artifacts will actually occur in the filtered sound because of the sharp filter cutoff - for more insight into this, revisit to the final lecture of the course.)

(d) (1 p.) $H_1 = 2 + 2e^{-j\hat{\omega}} = 2e^{-j\frac{\hat{\omega}}{2}}\left(e^{j\frac{\hat{\omega}}{2}} + e^{-j\frac{\hat{\omega}}{2}}\right) = 4e^{-j\frac{\hat{\omega}}{2}}\cos\frac{\hat{\omega}}{2}$. The corresponding magnitude plot is in Figure 8.

(e) (1 p.) $e^{j\hat{\omega}} = \cos\hat{\omega} + j\sin\hat{\omega}$.
$e^{-j\hat{\omega}} = \cos -\hat{\omega} + j\sin -\hat{\omega} = \cos\hat{\omega} - j\sin\hat{\omega}$.
$e^{j\hat{\omega}} - e^{-j\hat{\omega}} = \cos\hat{\omega} + j\sin\hat{\omega} - (\cos\hat{\omega} - j\sin\hat{\omega}) = 2j\sin\hat{\omega}$.
Hence, $\frac{e^{j\hat{\omega}} - e^{-j\hat{\omega}}}{2} = j\sin\hat{\omega}$.
$H_2 = 2 - 2e^{-j\hat{\omega}} = 2e^{-j\frac{\hat{\omega}}{2}}\left(e^{j\frac{\hat{\omega}}{2}} - e^{-j\frac{\hat{\omega}}{2}}\right) = 4je^{-j\frac{\hat{\omega}}{2}}\sin\frac{\hat{\omega}}{2}$.
$|H_2| = |4je^{-j\frac{\hat{\omega}}{2}}\sin\frac{\hat{\omega}}{2}| = |4\sin\frac{\hat{\omega}}{2}|$.
The corresponding magnitude plot is in Figure 9.

(f) (1 p.) The result is obtained by pointwise multiplication of $|H_1|$ and $|H_2|$. See Figure 10.

(g) (1 p.) Filters S_3 most strongly shows the desired bandpass characteristics with regard to the speech frequencies. Filter $|S_1|$ would emphasize the "hum" and attenuate higher frequencies in the speech signal. Filter $|S_2|$ would attenuate not just the "hum" but also lower frequencies in the speech signal.

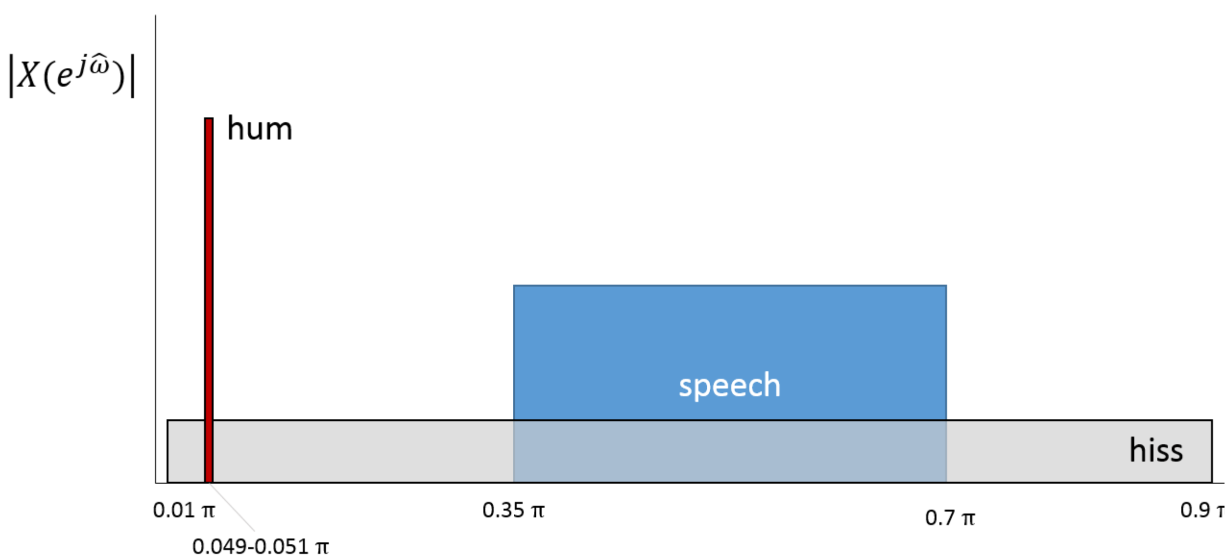

Figure 6: Plot corresponding to question 5(b).

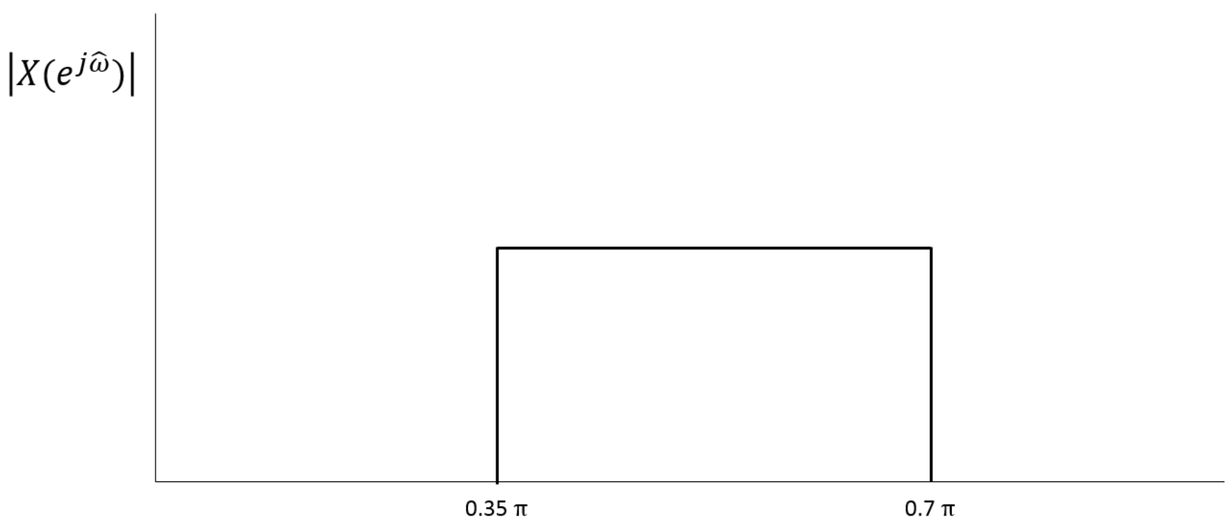

Figure 7: Plot corresponding to question 5(c).

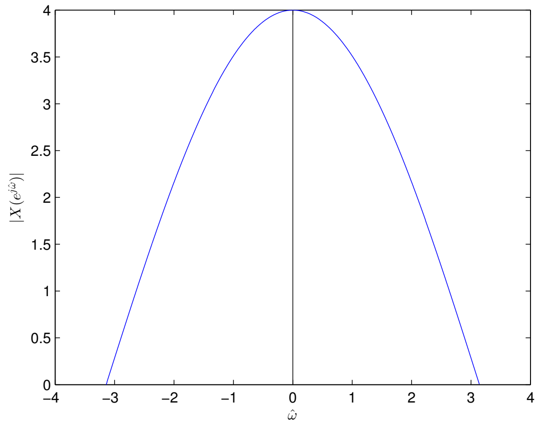

Figure 8: Plot corresponding to question 5(d).

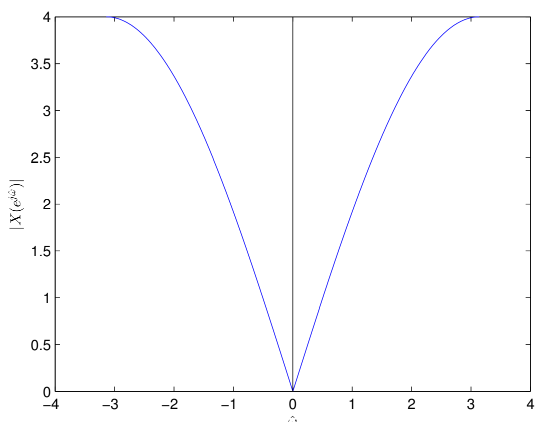

Figure 9: Plot corresponding to question 5(e).

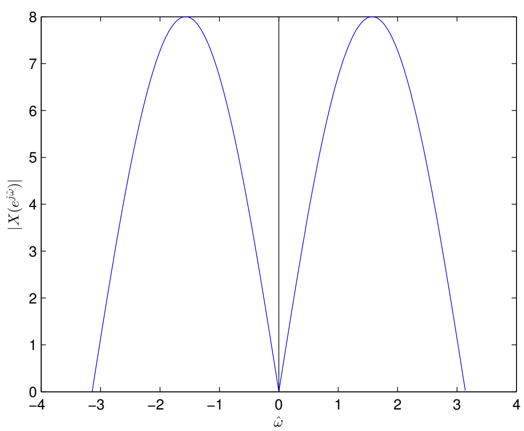

Figure 10: Plot corresponding to question 5(f).