Mid-term Exam Signal Processing TI2716-A

September 25$^{\text{th}}$, 2014
09:00 - 11:00 h

- This exam has four questions, for which a total of 27 points can be obtained.
- The allotted time for this exam is 2 hours.
- Use of the Equation Sheet TI2716-A is permitted.
- Please answer each question on a new sheet of paper.
- Questions may be answered in Dutch or English.

Question 1 (5 points total)

An input-output system $\mathcal{S}$ is given by

$$y[n] = x[n] - 5x[n-1] + 1.$$

(a) (1 p.) Show whether or not the system is causal.

(b) (2 p.) Show that the system is time-invariant.

(c) (2 p.) Use the superposition principle to show that the system is not linear.

Question 2 (9 points total)

Given is an LTI system $\mathcal{S}_1$ with the following impulse response:

$$h_1[n] = \delta[n] - 2\delta[n-1].$$

(a) (1 p.) Give the input-output relation (in terms of input $x[n]$ and output $y[n]$) for this system.

(b) (1 p.) $\mathcal{S}_1$ is an FIR filter. What is the order of the filter?

As input to $\mathcal{S}_1$, the following signal $x_1[n]$ is given:

n	≤ -2	-1	0	1	2	3	4	≥ 5
$x[n]$	0	0	1	2	2	1	0	0

(c) (1 p.) Write the input signal $x_1[n]$ as a sum of impulse signals $\delta[n]$.

(d) (2 p.) Determine output $y_1[n]$ by convolving $h_1[n]$ and $x_1[n]$.

Given is another LTI system $\mathcal{S}_2$ with the following impulse response:

$$h_2[n] = 2\delta[n] + \delta[n-2].$$

System $\mathcal{S}_1$ and $\mathcal{S}_2$ are put in cascade as indicated in Figure 1.

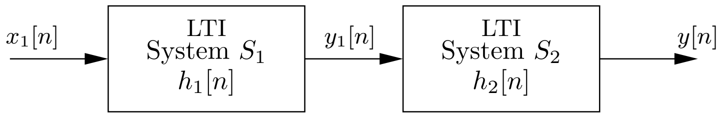

Figure 1: Cascaded system $\mathcal{S}_3$, composed of the LTI systems $\mathcal{S}_1$ and $\mathcal{S}_2$.

(e) (2 p.) Determine the overall impulse response $h[n]$ of the cascaded system $\mathcal{S}_3$.

(f) (2 p.) Determine the output $y[n]$ of this cascaded system for input signal $x_1[n]$.

Question 3 (7 points total)

A time-continuous sinusoidal signal is given by

$$x(t) = 2\cos(400\pi t - \pi/2).$$

(a) (2 p.) Sketch a plot of this signal for $t \in [0, 0.02 \text{ s}]$. What is the signal period, and what is the first $t > 0$ for which the signal reaches its maximum value?

(b) (1 p.) What is the Nyquist sampling rate of this signal?

$x(t)$ is sampled at $f_s = 800$ (Hz).

(c) (2 p.) Give the expression for the resulting discrete-time signal $x[n]$, and sketch a plot of that signal for $n \in [0, 20]$.

From this discrete-time signal $x[n]$, a continuous signal can be reconstructed again. Assume we can achieve perfect reconstruction, so reconstruction yields a sinusoidal signal.

(d) (2 p.) In case we reconstruct a continuous-time signal from $x[n]$ using a sampling frequency of 200 Hz, what is the frequency of the reconstructed signal, and what is the audible difference between this signal and the original signal $x(t)$?

Question 4 (6 points total)

(a) (1 p.) Express the complex number $z = -2 - 2\sqrt{3}j$ in the form of polar coordinates. The argument/phase should be between 0 and 2π.

(b) (1 p.) Express z in the form of a complex exponential. Again, the argument/phase should be between 0 and 2π.

(c) (1 p.) Write the complex exponential $w = 2e^{j\frac{\pi}{2}}$ in Cartesian form $a + bj$.

(d) (1 p.) Plot w and z on the complex plane. Please make separate plots for w and z.

(d) (2 p.) Calculate $z \times w$. Express the result both in Cartesian form, and as a complex exponential.