Signal Processing TI2716-A

Answers Partial exam 1

September 24, 2015, 13.30-15.30 h

Question 1 (9 points)

(a) (1 pt) The input-output relation for S_1 only consists of terms $x[n-n_0]$ with $0 \leq n_0$. As a consequence, the system only considers current and past samples, and therefore it is causal.

(b) (2 pt) According to the superposition principle, if we consider $y_A[n] = \sum_{k=0}^{2} \frac{1}{k+1} x_A[n-k]$ and $y_B[n] = \sum_{k=0}^{2} \frac{1}{k+1} x_B[n-k]$, then it should hold that

$$\sum_{k=0}^{2} \frac{1}{k+1} (\alpha x_A[n-k] + \beta x_B[n-k]) = \alpha\, y_A[n] + \beta y_B[n]\,.$$

This is indeed the case:

$$\sum_{k=0}^{2} \frac{1}{k+1} (\alpha x_A[n-k] + \beta x_B[n-k]) = \sum_{k=0}^{2} \frac{1}{k+1} \alpha x_A[n-k] + \sum_{k=0}^{2} \frac{1}{k+1} \beta x_B[n-k]$$

$$= \alpha \sum_{k=0}^{2} \frac{1}{k+1} x_A[n-k] + \beta \sum_{k=0}^{2} \frac{1}{k+1} x_B[n-k] = \alpha\, y_A[n] + \beta y_B[n].$$

(c) (2 pt) If the system is time-invariant, calculating the output $w[n]$ of S_1 for a delayed input signal $x[n-n_0]$ should yield the same output als calculating the output $z[n]$ of S_1 by calculating $y[n]$ for input $x[n]$, and then delaying $y[n]$ by n_0 samples.

$w[n] = \sum_{k=0}^{2} \frac{1}{k+1} x[(n-k)-n_0]$

$z[n] = \sum_{k=0}^{2} \frac{1}{k+1} x[(n-n_0)-k]$

Conclusion: $w[n] = z[n]$ so the system is time invariant.

(d) (2 pt) Again, we have to show whether the superposition principle holds (alternatively, a shorter proof can be given by showing whether the homogeneity principle holds, but here we will give the 'full' superposition proof.

If we give input $\alpha x_A[n] + \beta x_B[n]$ as input to system S_2, the output is $n - 2(\alpha x_A[n-1] + \beta x_B[n-1])$. If this output is equivalent to $\alpha\, y_A[n] + \beta y_B[n]$, the system is linear.

However, $\alpha y_A[n] + \beta y_B[n] = \alpha\,(n - 2x_A[n-1]) + \beta(n - 2x_B[n-1]) = \alpha n - 2\alpha x[n-1] + \beta n - 2\beta x_B[n-1]$. This is a different output than the answer given above, so the system is **not** linear.

(e) (2 pt) Again, we compute $w[n]$ and $z[n]$:

$w[n] = n - 2x[n-1-n_0]$ (when shifting input samples, only $x[n]$ is affected)

$z[n] = y[n-n_0] = (n-n_0) - 2x[(n-n_0)-1] \neq w[n]$

So the system is **not** time-invariant.

Question 2 (8 points)

(a) (1 pt) $h_3[n] = 2\delta[n] - \frac{1}{2}\delta[n-1] - \frac{1}{2}x[n-2]$.

(b) (2 pt)

n	0	1	2	3	4	≥5
$x_1[n]$	2	4	0	2	0	0
$h_3[n]$	2	-1/2	-1/2	0	0	0
$h_3[0]x_1[n]$	4	8	0	4	0	0
$h_3[1]x_1[n-1]$	0	-1	-2	0	-1	0
$h_3[2]x_1[n-2]$	0	0	-1	-2	0	-1
$h_3[n] * x_1[n]$	4	7	-3	2	-1	-1

So $y_1[n] = 4\delta[n] + 7\delta[n-1] - 3\delta[n-2] + 2\delta[n-3] - \delta[n-4] - \delta[n-5]$.

(c) (1 pt) see Figure 1.

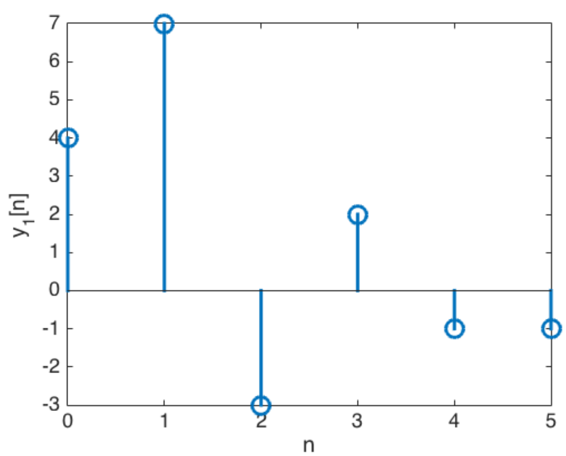

Figure 1. Plot for $y_1[n]$.

(d) (2 pt) As system S_3 is LTI, because of the superposition principle, the output of system S_3 for input signal $x_2[n]$ must be

$$y_2[n] = 2y_1[n] + y_1[n-1]$$

$$= 2 * (4\delta[n] + 7\delta[n-1] - 3\delta[n-2] + 2\delta[n-3] - \delta[n-4] - \delta[n-5])$$

$$+4\delta[n-1] + 7\delta[n-2] - 3\delta[n-3] + 2\delta[n-4] - \delta[n-5] - \delta[n-6]$$

$$= 8\delta[n] + 18\delta[n-1] + \delta[n-2] + \delta[n-3] - 3\delta[n-5] - \delta[n-6].$$

(e) (1 pt) $h_4[n] = 2\delta[n-2] - \frac{1}{2}\delta[n-3] - \frac{1}{2}x[n-4]$.

(f) (1 pt) $h_A[n] = \delta[n-2]$ (a system with impulse response $\delta[n-k]$ will delay an input signal $x[n]$ by k samples).

Question 3 (9 points)

(a) (1 pt) For the signal plot, see Figure 2. The first $t > 0$ for which this signal reaches its maximum value is $t = 0.0075$.

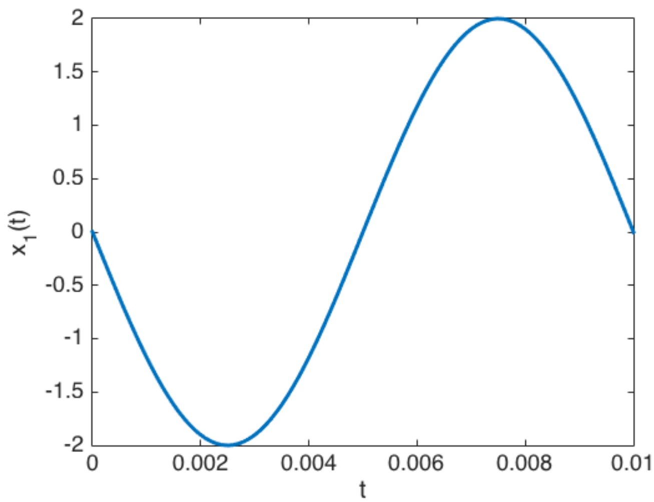

Figure 2. Plot for $x_1(t)$.

(b) (1 pt) The frequency of $x_1(t)$ is 100 Hz ($200\pi t = 2\pi f t$). Hence, the Shannon-Nyquist sampling rate is $2 \cdot 100 = 200\text{ Hz}$.

(c) (1 pt) $x_1(t) = 2\cos\left(200\pi t + \frac{\pi}{2}\right)$.

*There seems to have been a linguistic misunderstanding with many students here: by 'please depart from your answer', we actually intended you to *use* your answer as the basis for the further questions.*

(d) (2 pt) $x_1[n] = 2\cos\left(\frac{\pi}{4}n + \frac{\pi}{2}\right)$. The plot is displayed in Figure 3.

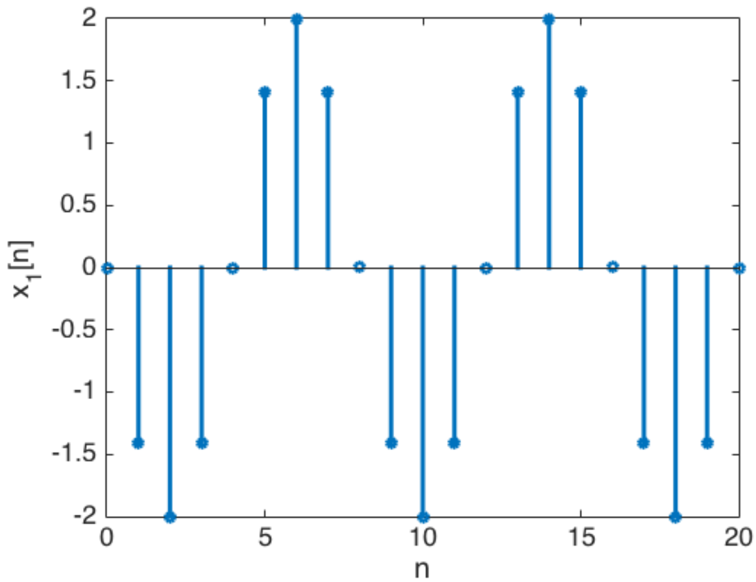

Figure 3. Plot for $x_1[n]$.

(e) (1 pt) $x_1[n] = 2\cos\left(\frac{\pi}{4}n + \frac{\pi}{2}\right) = \frac{1}{2} \cdot 2e^{j\left(\frac{\pi}{4}n+\frac{\pi}{2}\right)} + \frac{1}{2} \cdot 2e^{-j\left(\frac{\pi}{4}n+\frac{\pi}{2}\right)} = e^{j\left(\frac{\pi}{4}n+\frac{\pi}{2}\right)} + e^{-j\left(\frac{\pi}{4}n+\frac{\pi}{2}\right)}$..

(f) (1 pt) $x_1[1] = e^{j\left(\frac{\pi}{4}1+\frac{\pi}{2}\right)} + e^{-j\left(\frac{\pi}{4}1+\frac{\pi}{2}\right)} = e^{j\left(\frac{3}{4}\pi\right)} + e^{-j\left(\frac{3}{4}\pi\right)}$.

So $z = x_1[1] \cdot e^{j\frac{3}{4}\pi} = \left(e^{j\frac{3}{4}\pi} + e^{-j\frac{3}{4}\pi}\right) \cdot e^{j\frac{3}{4}\pi} = e^{j\frac{6}{4}\pi} + e^{0}$

$= \left(\cos\left(\frac{3}{2}\pi\right) + j\sin\left(\frac{3}{2}\pi\right)\right) + 1 = -j + 1 = 1 - j$.

In case you took our given $x_1'[1]$ as alternative, you would have found the same answer. We had intended to ask $x_1'[1]$= $e^{j\frac{7}{4}\pi} + e^{-j\frac{7}{4}\pi}$(which would give outcome $w = -1 + j$) but for our indicated $x_1'[1]$= $e^{j\frac{5}{4}\pi} + e^{-j\frac{5}{4}\pi}$you will actually find that $w = z$.

(g) (2 pt) $x_2(t)$ can generally be expressed as $x_2(t) = 2\cos\left(100\pi t + \frac{\pi}{2}\right)$. $f_s = 400$ Hz, so the time between consecutive $x_1[n]$ samples is considered to be $\frac{1}{400} = 0.0025$ s. In $x_1[n]$, the digital frequency was $\pi/4$, so one period of the signal consists of 8 samples. Hence, one period of $x_2(t)$ will be $8 \cdot 0.0025 = 0.02$ s.

A final thing to consider is that **we apply linear interpolation**, and with only 8 samples in one signal period to render to continuous time, this will have visible effect on the time-continuous signal. One period of $x_2(t)$ is plotted in Figure 4.

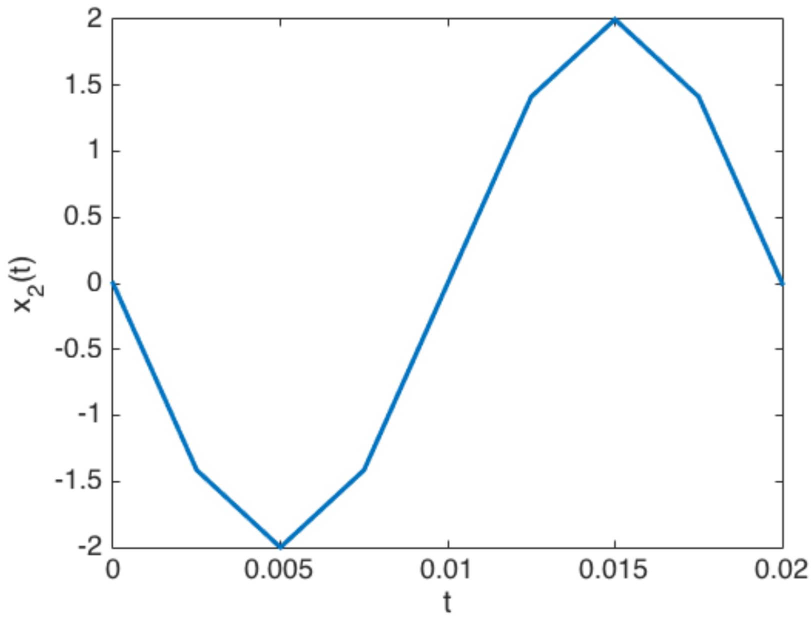

Figure 4. Plot for $x_2(t)$.