Signal Processing TI2716-A

Partial exam 1

September 24, 2015, 13.30-15.30 h

- This exam has 3 questions, for which a total of 26 points can be obtained.
- The allotted time for this exam is 2 hours.
- Use of the Equation Sheet TI2716-A, including any own hand-written notes on the printed sheet, is permitted.
- Use of a calculator is not permitted.
- For each of the 3 questions, start your answer on a new page.
- Questions may be answered in Dutch or English.

Question 1 (9 points)

We consider the system S_1 that is characterized by the following input-output relation:

$$S_1: \qquad y_1[n] = \sum_{k=0}^{2} \frac{1}{k+1} x[n-k]$$

(a) (1 pt) Explain if S_1 is causal or not.

(b) (2 pt) Use the **superposition principle** to prove whether system S_1 is linear or not**.**

(c) (2 pt) Prove whether system S_1 is time-invariant or not.

We next consider the system S_2 that is characterized by the following input-output relation:

$$S_2: \qquad y_2[n] = n - 2x[n-1]$$

(d) (2 pt) Prove whether system S_2 is linear or not.

(e) (2 pt) Show that system S_2 is **not** time-invariant.

Question 2 (8 points)

We consider an LTI system S_3, characterized by the following input-output relation:

$$S_3: \qquad y_3[n] = 2x[n] - \frac{1}{2}x[n-1] - \frac{1}{2}x[n-2]$$

(a) (1 pt) What is the impulse response $h_3[n]$ of this system?

An input signal $x_1[n]$ is specified as follows:

n	≤ -1	0	1	2	3	4	≥ 5
$x_1[n]$	0	2	4	0	2	0	0

(b) (2 pt) Calculate the output of system S_3 for this given input $x_1[n]$ by applying direct convolution.

(c) (1 pt) Make a time versus amplitude plot (with time on the x-axis) of the output signal calculated for question (b).

We consider another input signal $x_2[n]$, which is based on signal $x_1[n]$ as follows:

$$x_2[n] = 2x_1[n] + x_1[n-1]$$

(d) (2 pt) Using **only the result of question (b) and the fact that system S_3 is LTI**, show that the output of system S_3 for input signal $x_2[n]$ is

$$y_2[n] = 8\delta[n] + 18\delta[n-1] + \delta[n-2] + \delta[n-3] - 3\delta[n-5] - \delta[n-6].$$

Now consider a black-box system S_4, of which we only can observe input and output. When giving an input signal $x[n]$ to both S_3 and S_4, we observe that the output of system S_4 is identical to the output of system S_3, but delayed over two samples.

(e) (1 pt) Give the impulse response of system S_4.

Following the above observation, we can consider S_4 as a system that puts system S_3 in cascade with another system S_A.

(f) (1 pt) What is the impulse response $h_A[n]$ of system S_A?

Question 3 (9 points)

We consider a time-continuous sinusoidal signal $x_1(t)$ specified as

$$x_1(t) = -2\sin(200\pi t)$$

(a) (1 pt) Sketch one period of this signal, starting at $t = 0$. What is the first $t > 0$ for which this signal reaches its maximum value?

(b) (1 pt) What is the Shannon-Nyquist sampling rate of $x_1(t)$?

We can rewrite the signal $x_1(t)$ in the form

$$A\cos(\omega t + \phi)$$

(c) (1 pt) Rewrite the expression for signal $x_1(t)$ as a cosine with $A \geq 0$ and $-\pi < \phi \leq \pi$.

For the remaining questions, please depart from the expression you gave for $x_1(t)$ under question (c).

We sample $x_1(t)$ at $f_s = 800$ Hz, obtaining a discrete-time signal $x_1[n]$.

(d) (2 pt) Give the expression for $x_1[n]$, and sketch a plot of the signal for $n \in [0,20]$.

(e) (1 pt) Express signal $x_1[n]$ as a sum of complex exponentials.

(f) (1 pt) Let $z = x_1[1] \cdot e^{j\frac{3}{4}\pi}$. Show that z can be expressed in Cartesian form as $1 - j$.

NOTE: *If you could not solve (d) and (e), take $x_1'[1] = e^{j\frac{5}{4}\pi} + e^{-j\frac{5}{4}\pi}$ and express the product $w = x_1'[1] \cdot e^{j\frac{3}{4}\pi}$ in Cartesian form as an alternative answer to question (f). ~~Note that $w \neq 1 - j$~~.*

We now want to convert signal x_1[n] into a time-continuous signal again. This is done by employing linear interpolation, and taking $f_s = 400$ Hz. We call the resulting signal $x_2(t)$.

(g) (2 pt) Sketch one period of signal $x_2(t)$, starting at $t = 0$.