Delft University of Technology
Calculus for CSE (CSE1200)
Test 3, 28-1-2019, 9:00 – 12:00

Short Answer Questions

Remarks: No calculators allowed, grade $= \frac{\text{Total score}}{9} + 1$.

4pt 1. The function $f(x) = e^{3x}$ can be written as power series near $x = 1$:
$e^{3x} = c_0 + c_1(x-1) + c_2(x-1)^2 + \ldots$. Find c_0, c_1 and c_2.

$c_0 =$ [ ] $c_1 =$ [ ] $c_2 =$ [ ]

Answer:
Use the definition of the Taylor series:

$$f(a) + f'(a)(x-a) + \frac{1}{2}f''(a)(x-a)^2 + \ldots$$

Apply this with $a = 1$ and $f(x) = e^{3x}$. Note that $f'(x) = 3e^{3x}$ and $f''(x) = 9e^{3x}$. Plugging in we find:

$$e^{3x} = e^3 + 3e^3(x-1) + \frac{9}{2}e^3(x-1)^2 + \ldots$$

2pt 2. Write the following complex number in the form $a + bi$, with $a, b \in \mathbf{R}$:

$\frac{2i}{3-i} =$ [ ]

Answer:
We have:

$$\frac{2i}{3-i} = \frac{2i(3+i)}{(3-i)(3+i)} = \frac{-2+6i}{10} = -\frac{1}{5} + \frac{3}{5}i.$$

2pt 3. Find all complex solutions to the equation $z^3 = -2 + 2i$.
Leave your answer(s) in polar form.

[ ]

Answer:
Note that $-2 + 2i = \sqrt{8}e^{\frac{3}{4}\pi i}$. Writing $z = re^{i\theta}$ we find that $r^3 = \sqrt{8}$ and $3\theta = \frac{3}{4}\pi i + 2k\pi$ with $k \in \mathbb{Z}$. We find that $r = \sqrt{2}$ and $\theta = \frac{1}{4}\pi + \frac{2}{3}k\pi$. This gives solutions:

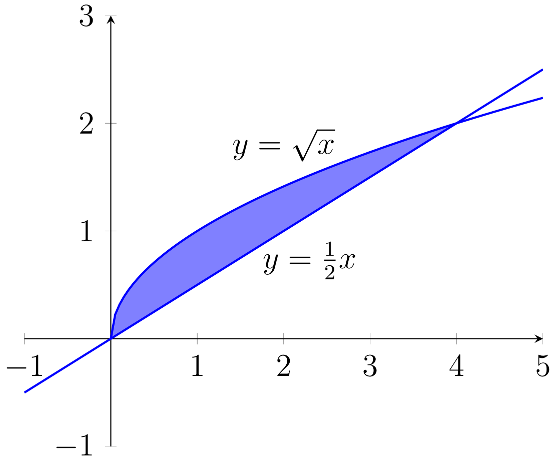
$$z_1 = \sqrt{2}e^{\frac{1}{4}\pi i}$$
$$z_2 = \sqrt{2}e^{\frac{11}{12}\pi i}$$
$$z_3 = \sqrt{2}e^{\frac{19}{12}\pi i}$$

4pt 4. Let $D \subset \mathbf{R}^2$ be the bounded region bounded by the lines $y = \frac{1}{2}x$ and $y = \sqrt{x}$. Find the correct limits (note the order of integration!).

$$\iint_D f(x,y)\,dA = \int_{\boxed{}}^{\boxed{}} \int_{\boxed{}}^{\boxed{}} f(x,y)\,dx\,dy.$$

Answer:

Note that the lines intersect in $(0,0)$ and $(4,2)$. Sketch of D:



The correct limits are:

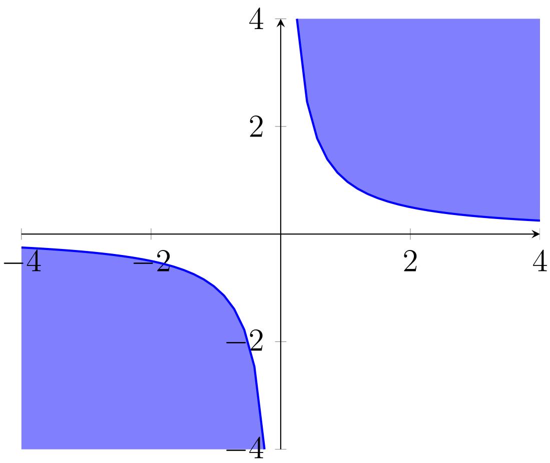
$$\int_0^2 \int_{y^2}^{2y} f(x,y)\,dx\,dy.$$

4,2,2,2pt

5. Consider the following function: $f(x,y) = \sqrt{xy-1}$.

a. Sketch the maximal domain of this function. Clearly indicate which parts belong to the domain and which do not.

Answer:
The maximal domain is given by $D = \{(x,y) : xy \geq 1\}$. The line $xy = 1$, or equivalently, $y = \frac{1}{x}$ is a hyperbola. We obtain the following sketch:



b. Find $\nabla f(x,y)$

Answer:

$$\nabla f(x,y) = \left\langle \frac{y}{2\sqrt{xy-1}}, \frac{x}{2\sqrt{xy-1}} \right\rangle.$$

c. Find the linearization of f at the point $(2,5)$

Answer:
We have

$$L(x,y) = f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5)$$

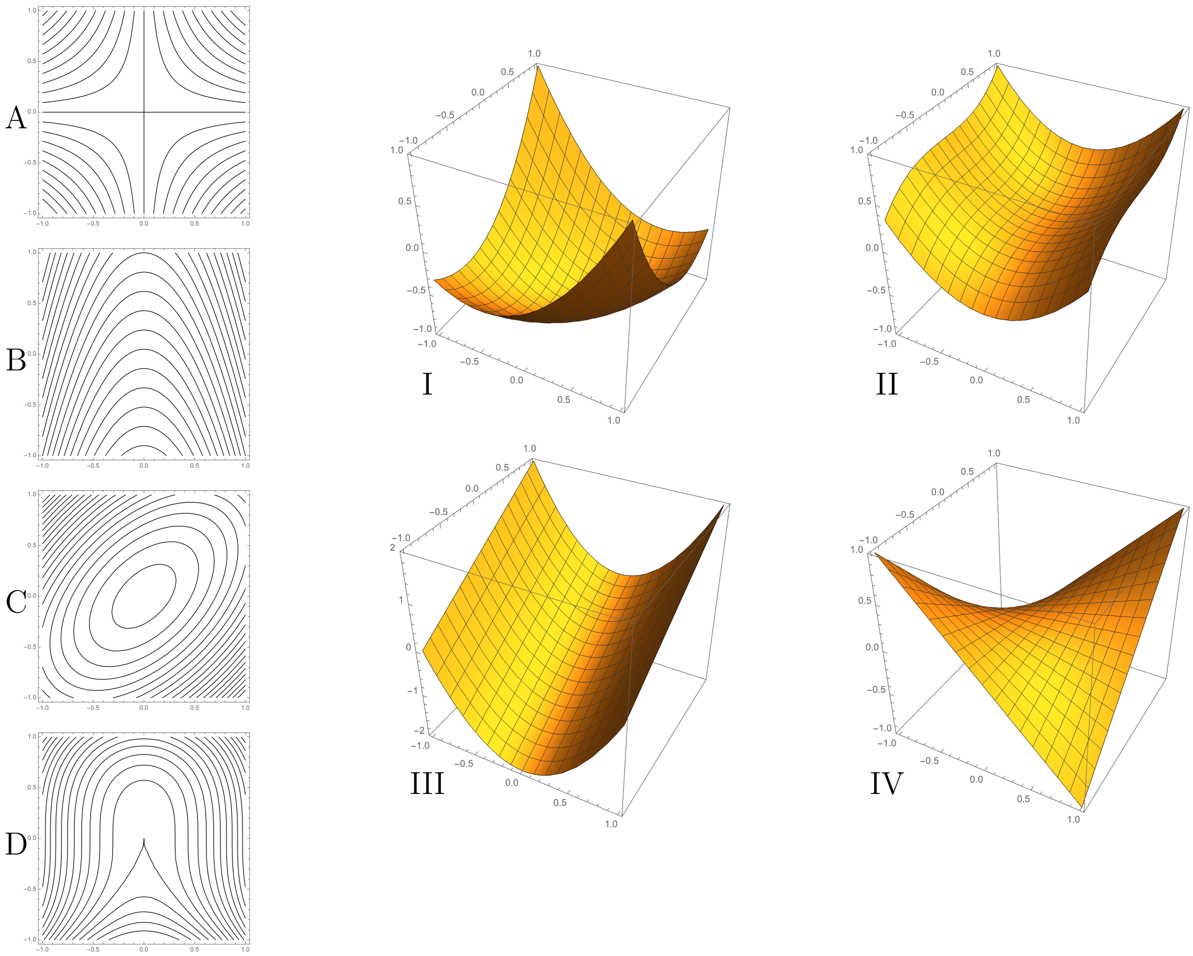
$$= 3 + \frac{5}{6}(x-2) + \frac{1}{3}(y-5).$$

d. Find the *minimal* value of the directional derivative at the point $(2,5)$.

Answer:
The minimal value is $-|\nabla f(2,5)| = -\sqrt{(\frac{5}{6})^2 + (\frac{1}{3})^2} = -\frac{1}{6}\sqrt{29}$.

4pt

6. Match the following graphs of four functions with their graphs of level curves. You do not have to give an explanation.



A: B: C: D:

Answer:

$$A \to IV$$
$$B \to III$$
$$C \to I$$
$$D \to II$$