Endterm Calculus CSE1200

1 February 2023, 09:00–12:00

READ CAREFULLY: The following exam consists of 4 short answer questions and 5 open questions. Note your answer on the separate exam paper for both kinds of questions and make sure to clearly add your name and student number on that paper. You only hand in this separate exam paper. You are not allowed to use a calculator (it will also not be necessary). You may find a copy of the formula sheet on the final page of this exam paper.

You don't need to provide decimal expansions or any level of simplification beyond what is specifically required in the question. This means that answers like $\frac{\sqrt[8]{3}}{4^4}$ and $\pi^2 \cos\left(\frac{3^{\frac{1}{3}}}{2\pi}\right)$ are perfectly acceptable.

For the short answer questions an argumentation/computation is not necessary and will not be taken into account, only the final answer will be graded.

For the open questions an answer without argumentation/computation will be incorrect. The best practice for writing your argumentation/computation is to use actual sentences. So the answer "The equation $x + 1 = 2$ implies that $x = 2 - 1 = 1$" is better than simply answering "$x + 1 = 2 \implies x = 2 - 1 = 1$". Both answers would get full points, but only the first one gets happy graders/teachers.

Grade: The points obtainable for each question are noted next to the question and your grade is computed as

$$\text{Grade} = \frac{\text{Obtained Points}}{5} + 1$$

Short Answer Questions

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3 pts **1.** Compute the angle between the vectors

$$\mathbf{v} = \langle 1, 0, 1\rangle \quad \text{and} \quad \mathbf{w} = \left\langle \sqrt{8} + \sqrt{12},\ \sqrt{24},\ \sqrt{8} - \sqrt{12}\right\rangle$$

3 pts **2.** Find all complex numbers z such that $z^3 = 8i$, give your answer(s) in the form $z = a + bi$.

√

3. Consider the function $f(x,y) = x\cos(\pi y) - xy$ at the point $P = (1,1)$

3 pts a) Find a unit vector in the direction of steepest descent.
(Steepest descent means going down fastest)

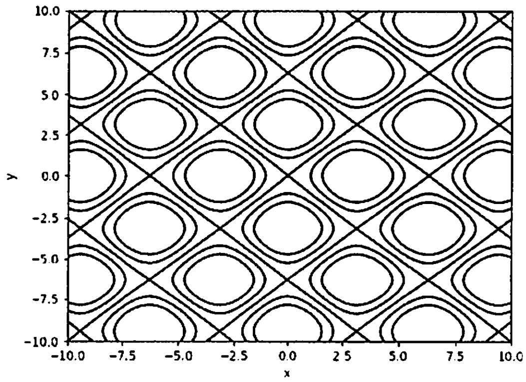
2 pts b) Find the directional derivative in the direction of steepest descent.

√

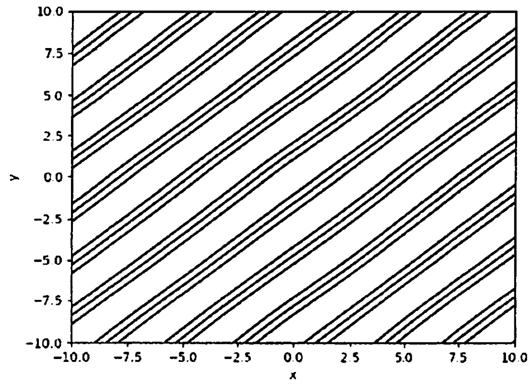
4. Match the function with the correct contour plot

2 pts a) $f(x,y) = \cos(x - y)$

2 pts b) $g(x,y) = xy^2$



A



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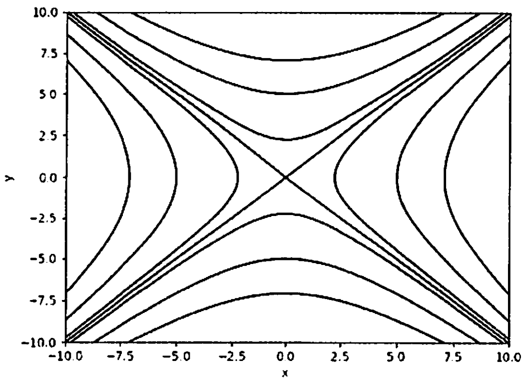

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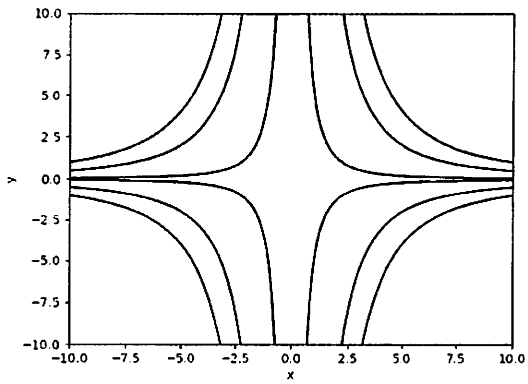

D

Open Questions

Provide computations and argumentations!

1. Consider the power series

$$\sum_{n=0}^{\infty} \frac{n}{2^n(n^2+1)}(x-1)^n$$

✓ 4 pts a) Find the interval of convergence.

~ 2 pts b) Let $g(x)$ denote the function represented by the power series on the interval of convergence. Compute $g^{(5)}(1)$.

2. Consider the function

$$f(x,y) = xy - x - y^2 + 1$$

on the domain D given by the closed square bounded by the lines $x = 0$, $x = 3$, $y = 0$ and $y = 3$, i.e.

$$D = \{(x,y) \mid 0 \leq x \leq 3 \text{ and } 0 \leq y \leq 3\}.$$

✓ 3 pts a) Find the critical points of f in the interior of D and classify each point as a local maximum, local minimum or saddle point.

✓ 2 pts b) Find the absolute maximum and absolute minimum of f on D.

✓

3. Consider the function $g(x,y) = \cos(x^2)$ on the domain D given by the interior of the triangle with vertices $(0,0)$, $(2,0)$ and $(2,2)$.

5 pts a) Compute $\displaystyle\iint_D g(x,y)\,dA$.

2 pts b) Let $G(x) = \displaystyle\int_2^x \cos(t^2)\,dt$ and compute $\displaystyle\int_0^2 G(x)\,dx$.

(*make sure to argue why the result is correct*)

4 pts

4. Consider the function $f(x) = e^{-x^2}$.

a) Give the Taylor series centered at 0 for an anti-derivative of $f(x)$.

2 pts

b) Approximate $\int_0^{0.1} f(x)\,dx$ with an error below $10^{-7} = 0.0000001$.

(make sure to argue why the approximation is accurate enough)

5. Consider the sequence of complex numbers z_n given by

$$z_{n+1} = \left(\frac{1}{2} + \frac{1}{2}i\right) z_n \quad \text{and} \quad z_0 = \frac{1}{2} + \frac{1}{2}i.$$

3 pts

a) Give z_0, z_1 and z_2 in polar form or polar exponential form.

2 pts

b) Give a formula for z_n in polar form or polar exponential form.

1 pt

c) Compute $\lim_{n\to\infty} |z_n|$.

(make sure to argue why your answer is correct)