Exam EE1510AM part I

Electricity and Magnetism

Wednesday, March 15, 2023, 9:00-11:00 a.m.

This exam consists of 2 pages with 2 assignments.

The total number of credits is 90.

The number of credits rated for each assignment is listed to the left of each assignment.

Start every assignment on a new sheet and write on every sheet of each worked out assignment your name and student number.

45 punten

Opgave 1

Consider a very thin disc in the xy-plane with its center positioned at $< 0, 0, 0 >$ and radius $r = R$. The disc contains a constant surface charge density $\sigma = \sigma_0$.

a.) Determine the total charge Q on the disc.

The electric field $\mathbf{E}$ that is excited by the surface charge density σ on the disc is determined at the position P in $< 0, 0, z >$.

b.) Show that the electric field $\mathbf{E}(0,0,z)$ is:

$$\mathbf{E}(0,0,z) = \frac{\sigma_0}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{R^2+z^2}}\right)\mathbf{i}_z$$

c.) Derive from the expression in **b.)** the electric field $\mathbf{E}(0,0,z)$ when the disc is extended to an infinite plane with surface charge density σ.

d.) Show that the potential $V(0,0,z)$ due to the surface charge density σ on the disc is:

$$V(0,0,z) = \frac{\sigma_0}{2\epsilon_0}(\sqrt{R^2+z^2} - z),$$

and show that $V(0,0,z) \to 0$ when $z \to \infty$.

Suppose the disc with radius $r = R$ is enclosed by a perfectly conducting, thin spherical shell with radius $r = 2R$ and center at $< 0, 0, 0 >$.

e.) Determine in this case the electric field $\mathbf{E}$ when $r > 2R$.

45 punten

Opgave 2

Consider two concentric spheres with their centers located at $< 0, 0, 0 >$, where the inner sphere is a solid perfectly conducting sphere with radius $r = R$, the outer sphere is a very thin perfectly conducting shell with radius $r = 2R$.
The space between the inner sphere with radius $r = R$ and the outer sphere with radius $r = 2R$ contains a volume charge density $\rho = k_0 R^2/r^2$.
On the outside of the outer sphere with radius $r = 2R$ we have no charge.

√ **a.)** Determine the electric field $\mathbf{E}$ for all r, with $0 < r < \infty$.

We assume that the potential $V(r) = 0$, when $r \to \infty$.

b.) Determine the potential $V(r)$ for all r, with $0 < r < \infty$.

c.) Give an expression for the capacitance C formed by the perfectly conducting spheres with radius $r = R$ and $r = 2R$.

In addition to the two concentric spheres described in the configuration above, we now add a very long cylinder with radius $s = R$ that contains a volume charge density $\rho(s) = k_0 s/R$. The axis of the cylinder points in the z-direction and is positioned at $< 4R, 0, z >$.

√ **d.)** Determine the electric field $\mathbf{E}_{\text{cylinder}}(x, 0, z)$ that is generated by the volume charge density ρ inside the cylinder, when $0 < x < 3R$ and when $3R < x < 4R$.

√ **e.)** Find an expression for the total electric field $\mathbf{E}_{\text{total}} = \mathbf{E}_{\text{cylinder}} + \mathbf{E}_{\text{spheres}}$ in the point $< 3R/2, 0, 0 >$.

End of Exam