Practice midterm CSE1210/EE1M31

1. The following probabilities about two events A and B are known:
   $P(A|B) = 0.8, \qquad P(B^c|A^c) = 0.75, \qquad P(A \cap B) = 0.2.$

   (a) Determine $P(A)$.

   (b) Are A and A^c independent?

2. Suppose vaccination is being used as a way to protect people from getting ill from a certain disease. 80% of the population has been vaccinated twice, 5% has been vaccinated once, the remaining 15% has not been vaccinated. People who have been vaccinated twice have a 1% probability of getting ill, people who have been vaccinated only once have a 10% probability of getting ill, people who have not been vaccinated have a 40% probability of getting ill. What is the probability that a random ill person, has been vaccinated twice? You do not need to simplify your answer.

3. The random variables X_1, X_2 and X_3 each have a distribution belonging to one of the families we have seen. Give the three distributions including the value of the parameter(s), i.e. your answer could be something like $X_1 \sim \text{Pois}(10)$.

   (a) A scratchcard contains 4 symbols. Each of the four is chosen from the set "circle, triangle, square, octagon"where all shapes have the same probability (shapes can be repeated on a single card). A scratchcard is winning if the four symbols are identical. Let X_1 be the number of winning scratchcards in a set of 10.

   (b) $X_2 = 2 - Y - Z$ where $Y \sim \text{Ber}(0.3)$ and $Z \sim \text{Ber}(0.3)$ are independent random variables.

   (c) $X_3 = 3W - 2$ where $W \sim U(5, 6)$.

4. Consider the random variable X with density function $f(x) = x + 1$ if $-1 \leq x \leq 0$ and $f(x) = -x + 1$ if $0 \leq x \leq 1$ (else $f(x) = 0$).

   (a) Compute $E[X]$.

   (b) Compute the 68%-percentile $q_{0.68}$.

   (c) Compute $E[X^2]$.

5. For each of the following functions, decide whether or not they are a probability density function. All functions are zero outside of the specified interval.

   (a) $f(x) = |x|$ if $-1 \leq x \leq 1$

   (b) $g(x) = \frac{1}{2}\sin(x)$ if $0 \leq x \leq 3\pi$

   (c) $h(x) = x$ if $1 \leq x \leq 2$

6. Two random variables X and Y have the properties $Var(Y) = 1$, $Var(X + 3Y) = 7$, $Cov(X, Y) = -2$. Compute $Var(X)$.

7. Consider two independent random variables X and Y, which give the results of two independent throws of fair 4-sided dice with faces 1, 2, 3, 4. Consider $U = |X - Y|$. Also consider V, where $V = 1$ if the product XY is even, $V = 0$ if XY is odd. The joint probability mass function is given in the table.

   (a) What are the values for a, b, c?

   (b) What is $F_U(1)$?

   (c) Are U and V independent?

	$U=0$	$U=1$	$U=2$	$U=3$
$V=0$	1/8	0	1/8	a
$V=1$	1/8	3/8	b	c

8. Consider the random variables X and Y with joint density function $f_{X,Y}(x,y) = 6x^{-3}y^{-4}$ if $x \geq 1$ and $y \geq 1$ (else the function is zero).

   (a) Compute the marginal density function for Y.

   (b) Compute $P(X = Y)$.

   (c) Express $P(X \leq 2 \text{ and } Y \leq 2)$ using integrals.

   (d) Express $E[(X - Y)]$ using integrals.

9. Consider 75 independent random variables X_1 to X_{75}, all having a $Gam(3,2)$ distribution. This is a distribution with density function $f(x) = \frac{1}{16}x^2e^{-x/2}$ and which has expectation equal to 6 and variance equal to 12. What is the probability that $A \geq 6.6$, where A is the average of these 75 random variables?

10. Chebyshev's inequality tells us that for each random variable Y with variance σ^2, we have that $P(|Y - E[Y]| \geq a) \leq \frac{\sigma^2}{a^2}$. Now consider a continuous random variable X for which $E[X] = 4/3$ and $E[X^2] = 145/81$ and for which you know that X cannot be less than 1. From Chebyshev's inequality we can obtain $P(X \geq 5/3) \leq b$. Find b.

11. When stars such as our Sun form, they do not always have the same radius: the radius is a random variable R. The stellar mass M is then also variable, since it depends on the radius. For a certain group of stars, this relation is given by $M = cR^{7/3}$ with c a constant. If $E[R] = \mu$, what can you then say about $E[M]$?

    (A) $E[M] > c\mu^{7/3}$ **(B)** $E[M] = c\mu^{7/3}$ **(C)** $E[M] < c\mu^{7/3}$
    (D) Impossible to tell without more information on the distribution of R

12. Consider the electrical circuit you find below the final question. There are three switches. Let S_1, S_2, S_3 all be Ber(1/2) random variables, where a 1 means that the switch is closed, letting the current through. The circuit is closed if the current can pass through (this does not necessarily imply that all switches are closed). Compute the probability that the circuit is closed in the following cases:

    (a) S_1, S_2, S_3 are independent.

    (b) $\rho(S_1, S_2) = \rho(S_2, S_3) = -1$ where ρ denotes the correlation coefficient.