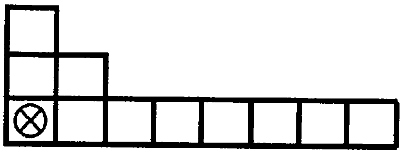
1 Chomp! The game involves removing squares from a rectangular board, say an $m \times n$ board. A move consists in taking a square and removing it and all squares to the right and above. Players alternate moves, and the person to take square (1, 1) loses. Think of a chocolate bar, where you eat what you break off, and piece (1,1) is poisonous. For example, starting with an 8 × 3 board, suppose the first player chomps at (3, 2) gobbling 12 pieces, and then second player chomps at (2, 3) gobbling 1 piece, leaving the following board, where ⊗ denotes the poisoned piece.



- A Show that this position is an N-position by finding a winning move for the first player.
- B Can you find two winning moves?
- C Prove that the rectangular $m \times n$ board is an N-position.

2 Three games. Solve the following zero-sum games.

A

$$\begin{pmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{pmatrix}$$

B

$$\begin{pmatrix} 1 & 4 & -1 & 5 & 2 \\ 4 & -1 & 5 & 1 & 2 \\ -1 & 5 & 1 & 4 & 3 \\ 5 & 1 & 4 & -1 & 3 \end{pmatrix}$$

C

$$\begin{pmatrix} 3 & 1 & 1 & 0 & 0 \\ 1 & 4 & 1 & 0 & 0 \\ 1 & 1 & 7 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 3 & 1 \end{pmatrix}$$

3 A recursive game. For $n = 0, 1, 2, \ldots$ the game G_n is given by

$$G_n = \begin{pmatrix} n+3 & n+2 \\ n+1 & G_{n+1} \end{pmatrix}$$

If the game continues forever, then Player I gets an infinite amount.

- A Prove that $n + 2 \leq V_n \leq n + 3$.
- B Determine V_n.
- C Suppose both players apply their optimal strategies. What is the probability that the game continues for more than two rounds?

4 Subgame perfect equilibrium. A PSE vector of strategies in a game in extensive form is said to be a subgame perfect equilibrium if at every vertex of the game tree, the strategy vector restricted to the subgame beginning at that vertex is a PSE. If a game has perfect information, a subgame perfect equilibrium may be found by the method of backward induction.

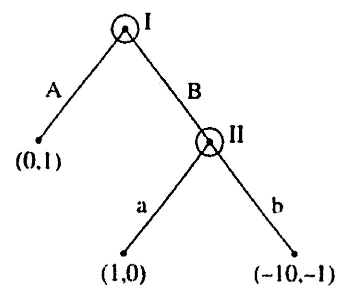


Fig. 2. An extensive form Game.

A Solve the game in Figure 2 for an equilibrium using backward induction.
B Put the game into strategic form and find another PSE of the strategic form game, and show that it is not subgame perfect.
C Find a cooperative strategy and the TU solution.

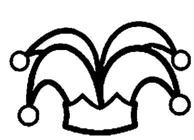
5 The Glove Market. Let N consist of two types of players, $N = P \cup Q$, where $P \cap Q = \emptyset$. Let the characteristic function be defined by

$$v(S) = \min\{|S \cap P|, |S \cap Q|\}$$

where $|P|$ denotes the number of players of P. The game (N, v) is called the *glove market* because of the following interpretation. Each player of P owns a right-hand glove and each player of Q owns a left-hand glove. If j members of P and k members of Q form a coalition, they have $\min\{j, k\}$ complete pairs of gloves, each being worth 1. Unmatched gloves are worth nothing.

A Suppose $|P| = 2$ and $|Q| = 2$. Determine the core.
B Determine the core if $|Q| > |P|$.
C Compute nucleolus and Shapley value if $|P| = 1$ and $|Q| = 3$.

Joker Rule



Traditionally, Game Theory exams come with a Joker. Each exercise has a maximum score of 6 points (all parts A,B,C count for 2). Your Joker exercise is worth: **half score + 3**. PUT YOUR JOKER ON YOUR WEAKEST EXERCISE! Your final grade is: (sum of the scores)/3.