Exam IN2405-A

Tuesday October 26th 2010

The problems are weighted equally in calculating the final grade. Therefore, try to spend your time wisely. Please, restrict yourself to the essence when answering discussion type of questions. You can answer in Dutch or English! Please, start every problem on a new sheet of paper and write down your name. Good luck!

Question 1

Consider the following input signal to an ideal C-to-D converter:

$$x(t) = (5 - 2\sin(2\pi 125t))\cos(2\pi 25t - \pi/2).$$

(a) Show, using phasors, that $x(t)$ can be expressed in the form

$$x(t) = A_1\cos(\omega_1 t + \phi_1) + A_2\cos(\omega_2 t + \phi_2) + A_3\cos(\omega_3 t + \phi_3),$$

and determine the values for the parameters A_k, ω_k, ϕ_k, $k = 1, 2, 3$.

Hint: Remember that $e^{j\pi} = -1$ *and* $e^{j\pi/2} = j$

(b) Sketch the two-sided spectrum of $x(t)$.

(c) Is the signal $x(t)$ periodic? If so, what is the period?

(d) Determine the minimum sampling rate that can be used to sample $x(t)$ without aliasing for any of the components.

(e) Let the sampling rate be $f_s = 350$ Hz. Determine the output $y(t)$ of the ideal D-to-C converter, and sketch its spectrum.

(f) Is the signal $y(t)$ periodic? If so, what is the period?

(g) Let the sampling rate be $f_s = 60$ Hz. Determine the output $y(t)$ of the ideal D-to-C converter, and sketch its spectrum.

(h) Is the signal $y(t)$ periodic? If so, what is the period?

Question 2

A linear time-invariant system is described by the difference equation

$$y[n] = x[n] + 2x[n-1] + x[n-2].$$

(a) Draw a block diagram that represents this system in terms of unit-delay elements, multipliers, and adders.

(b) Find the frequency response $H(e^{j\hat{\omega}})$.

(c) $H(e^{j\hat{\omega}})$ is a periodic function of $\hat{\omega}$; determine the period.

(d) Plot the magnitude and phase of $H(e^{j\hat{\omega}})$ as a function of $-\pi < \hat{\omega} < 3\pi$.

(e) Find all frequencies, $\hat{\omega}$, for which the ouput response to the input $e^{j\hat{\omega}n}$ is zero.

(f) When the input to the system is $x[n] = \sin(\pi n/10)$ determine the functional form for the output signal $y[n]$. *Hint: Make use of Euler's formula.*

Question 3

The two linear time-invariant systems in Fig. 1 are put in cascade; i.e., the output of the first system is the input to the second system, and the overall output is the output of the second system.

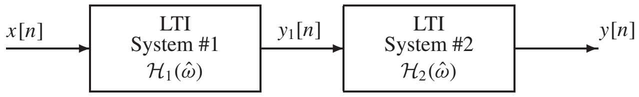

Figure 1: Cascade connection of two LTI systems.

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The two systems in Fig. 1 are both a 3-point moving averager, i.e.,

$$y_1 = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right)$$

and

$$y = \frac{1}{3}\left(y_1[n] + y_1[n-1] + y_1[n-2]\right)$$

(a) Determine the system functions $H_1(z)$ and $H_2(z)$, and the overall system function $H(z)$.

(b) Plot the poles and zeros of $H(z)$ in the complex z-plane. *Hint: Make use of the geometric series expansion and the N-th roots of unity.*

(c) Determine the impulse response $h[n]$ of the overall cascade system in Fig. 1 using the answer from Question (a).

(d) From $H(z)$, obtain an expression for the frequency response $H(e^{j\hat{\omega}})$ of the overall cascade system.

(e) Sketch the frequency response (magnitude and phase) functions of the overall cascade system for $-\pi \leq \hat{\omega} \leq \pi$. Make use of your result from question (d)

(f) Suppose that the input is

$$x[n] = 5 + 2\cos\left(\frac{2\pi}{3}n - \pi/4\right), \quad \text{for } -\infty < n < \infty$$

Obtain an expression for the output.

Question 4

Consider the following linear time-invariant (LTI) system with difference equation given by (Assume initial rest conditions):

$$y[n] = -0.7y[n-1] + 2x[n] - 3x[n-1].$$

(a) Determine the corresponding system function $H(z)$.

(b) Find the locations of the poles and zeros of this system. Is the system stable? (why/why not?)

(c) What is the impulse response $h[n]$ of this system?

Assume $x[n]$ is a suddenly applied input signal of the form

$$x[n] = 2e^{j\frac{\pi}{5}n}u[n],$$

where $u[n]$ is the unit step sequence.

(d) What is the z-transform $X(z)$ of the sequence $x[n]$? *Hint: Make use of the table of z-transforms.*

(e) Determine the z-transform $Y(z)$ of the corresponding output $y[n]$.

(f) Derive an expression for the ouput sequence $y[n]$ using the inverse z-transform. Indicate the transient part and the steady-state part of the output $y[n]$. *Hint: Make use of partial fraction expansion and make use of the table of z-transforms.*