Exam IN2405-A

Tuesday October 26th 2010

Question 1

(a)

$$\begin{aligned} x(t) &= \frac{5}{2}\left(e^{(2\pi 25t-\pi/2)j} + e^{-(2\pi 25t-\pi/2)j}\right) + \frac{1}{2}\left(e^{(2\pi 150t)j} - e^{-(2\pi 150t-\pi)j}\right) \\ &+ \frac{1}{2}\left(e^{(2\pi 100t+\pi)j} - e^{-(2\pi 100t)j}\right) \\ &= 5\cos(2\pi 25t-\pi/2) + \cos(2\pi 150t) - \cos(2\pi 100t) \qquad (1) \end{aligned}$$

(b) This follows from (a)

(c) The greatest common divisor of the three frequency components is $f =$ 25 Hz. The signal is therefore periodic with period $T = 1/25$.

(d) $f_s \geq 2f_{max} = 300$ Hz.

(e) Since $f_s > 2f_{max}$ we can conclude that $y(t) = x(t)$.

(f) See (c).

(g) Let $x_1(t) = 5\cos(2\pi 25t-\pi/2)$, $x_2(t) = \cos(2\pi 150t)$and $x_3(t) = -\cos(2\pi 100t)$. Since $f_s = 60$ Hz, we have that $y_1(t) = x_1(t)$. Further we have

$$x_2[n] = \cos(2\pi 150n/60) = \cos(2\pi n/2)$$

so that $y_2(t) = \cos(2\pi 30t)$. Also,

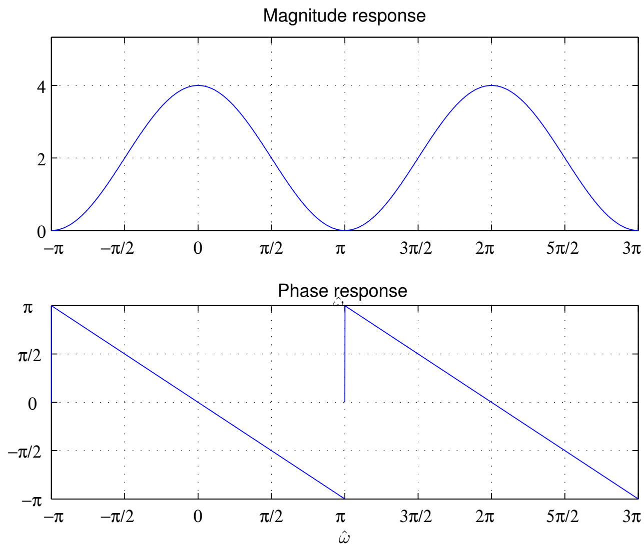
$$x_3[n] = -\cos(2\pi 100n/60) = -\cos(-2\pi 2n/6) = -\cos(2\pi n/3)$$

so that $y_3(t) = -\cos(2\pi 20t)$. Hence, $y(t) = 5\cos(2\pi 25t - \pi/2) + \cos(2\pi 30t) - \cos(2\pi 20t)$

Question 2

(b) The filter coefficients are $\{b_k\} = \{1, 2, 1\}$. Using $H(e^{j\hat{\omega}}) = \sum_{k=0}^{2} b_k e^{jk\hat{\omega}}$ we get $H(e^{j\hat{\omega}}) = 1 + 2e^{-j\hat{\omega}} + e^{-j2\hat{\omega}} = e^{-j\hat{\omega}}(2 + 2\cos(\hat{\omega}))$.

(c) $H(e^{j\hat{\omega}})$ is always 2π periodic. Proof: $H(e^{j\hat{\omega}+2\pi}) = e^{-j(\hat{\omega}+2\pi)}(2+2\cos(\hat{\omega}+2\pi)) = e^{-j\hat{\omega}}(2 + 2\cos(\hat{\omega})) = H(e^{j\hat{\omega}})$



(d)

(e) Determine these frequencies $\hat{\omega}$ where $|H(e^{-j\hat{\omega}})| = 0$. That is, $\cos(\hat{\omega}) = -1$. So $\hat{\omega} = \pi + k2\pi$ with k an integer.

Question 3

(a)

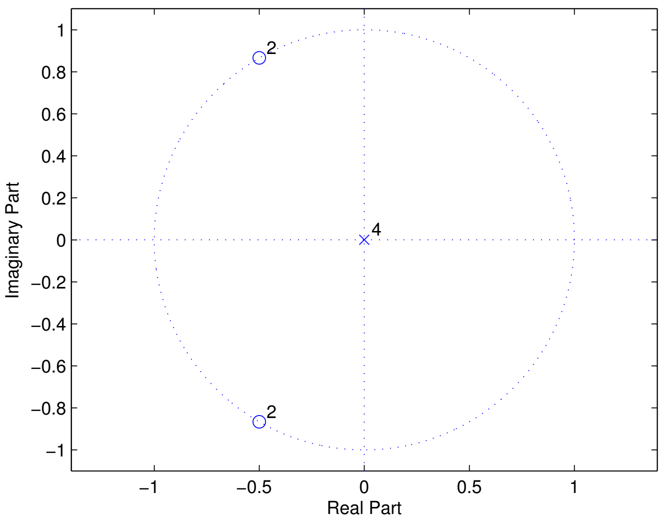
$$H_1(z) = H_2(z) = \frac{1}{3}(1 + z^{-1} + z^{-2})$$

$$H(z) = H_1(z)H_2(z) = \frac{1}{9}(1 + 2z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4})$$

(b)

$$H(z) = H_1(z)H_2(z) = \frac{1}{9}(1+z^{-1}+z^{-2})^2 = \frac{1}{9}\left(\frac{1 - z^{-3}}{1 - z^{-1}}\right)^2 = \frac{1}{9}\left(\frac{z^3 - 1}{z^3 - z^2}\right)^2$$

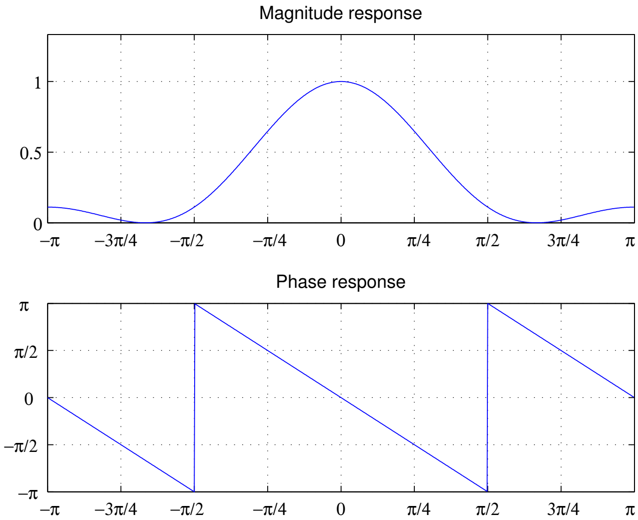
Poles: $(z^3 - 1)^2 = 0 \Longrightarrow 2$ x at $z = 1$, 2 x at $z = e^{j2\pi/3}$ and 2 x at $z = e^{j4\pi/3}$. Zeros:$(z^3 - z^2)^2 = 0 \Longrightarrow 4$ x at $z = 0$ and 2x at $z = 1$ The poles and zeros at $z = 1$ cancel, that means that we have 2 poles at $z = e^{j2\pi/3}$, 2 poles at $z = e^{j4\pi/3}$, and 4 zeros at $z = 0$.



(c) $h[n] = \frac{1}{9}(\delta[n] + 2\delta[n-1] + 3\delta[n-2] + 2\delta[n-3] + \delta[n-4])$

(d) $H(e^{j\hat{\omega}}) = e^{-j2\hat{\omega}} \frac{\sin^2(3\hat{\omega}/2)}{9\sin^2(\hat{\omega}/2)}$

(f) The system has a zero at $2\pi/3$. Therefore, $y[n] = 5$.



(e)

Question 4

(a) $H(z) = \frac{2-3z^{-1}}{1+0.7z^{-1}} = \frac{2z-3}{z+0.7}$

(b) zeros: z=3/2. Poles: z=-0.7

(c) $H(z) = \frac{2-3z^{-1}}{1+0.7z^{-1}} = \underbrace{\frac{2}{1+0.7z^{-1}}}_{part1} - \underbrace{\frac{3z^{-1}}{1+0.7z^{-1}}}_{part2}$

Using the table of z-transforms it follows for the inverse z-transform of part 1 is $h_1[n] = 2(-0.7)^n u[n]$ and of part 2 is $h_2[n] = -3(-0.7)^{(n-1)}u[n-1]$. The total impulse response is $h[n] = 2(-0.7)^n u[n] - 3(-0.7)^{(n-1)}u[n-1]$.

(d) Using the table of z-transform pairs we find $X(z) = \frac{2}{1-e^{j\pi/5}z^{-1}}$.

(e) $Y(z) = \frac{2-3z^{-1}}{1+0.7z^{-1}} \frac{2}{1-e^{j\pi/5}z^{-1}}$

(f) Partial fraction expansion:

$$Y(z) = \frac{A}{1+0.7z^{-1}} + \frac{B}{1-e^{j\pi/5}z^{-1}}$$

$$A = \frac{12\frac{4}{7}}{1 + e^{j\pi/5}\frac{10}{7}}$$

$$B = \frac{4 - 6e^{-j\pi/5}}{1 + 0.7e^{-j\pi/5}}$$

Using the inverse z-transform of each we get

$$y[n] = \underbrace{A(-0.7)^n u[n]}_{transient} + \underbrace{Be^{j\pi n/5} u[n]}_{steady\ state}$$