DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics, and Computer Science
Department of Mediamatics
Multimedia Signal Processing Group

SIGNAL PROCESSING (IN2405 - A)

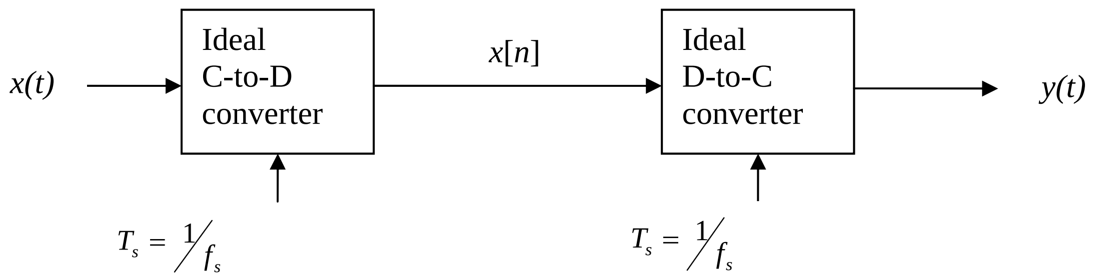
Written examination, Friday 22 January 2010 (14:00 - 17:00)

The problems are weighted equally in calculating the final grade. Therefore, try to spend your time wisely. Please, restrict yourself to the essence when answering "discussion type" of questions. You can answer in Dutch or English!

Please, start every problem on a **new** sheet of paper and write down your **name**. Good luck!

This exam contains 4 problems

Problem 1



Consider the above system:

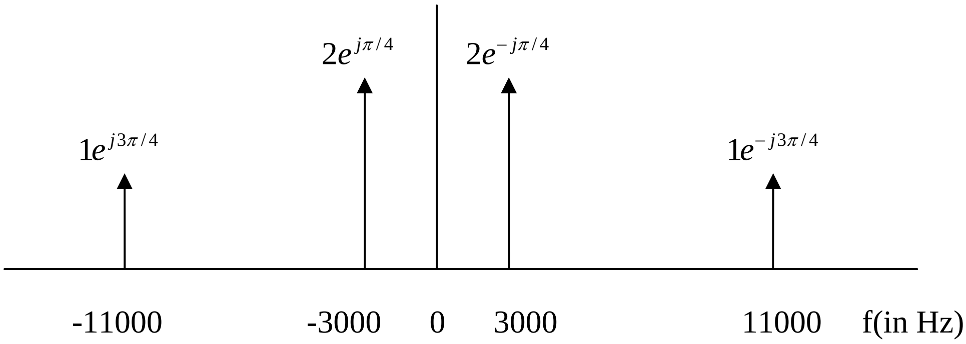
a) Suppose that the discrete-time signal $x[n]$ is given by the formula

$$x[n] = 10\cos\left(0.18\pi n + \frac{\pi}{4}\right)$$

If the sampling rate f_s = 2500 samples/sec; determine two *__different__* continuous-time signals values of $x(t) = x_1(t)$ and $x(t) = x_2(t)$ that could have been inputs to the above system: i.e., find $x_1(t)$ and $x_2(t)$ such that $x[n] = x_1(nT_s) = x_2(nT_s)$ if $T_s = \frac{1}{2500}$. Both of these input signals should have a frequency *__less__* than 2500 Hz. Give a formula for each signal.

b) For $x[n]$ given in part (a), what is the frequency of the analog signal $y(t)$ that will be reconstructed by the ideal D-to-C converter operating at sampling rate 2500 samples/second?

c) If the input signal $x(t)$ is given by the two-sided spectrum representation shown below, determine a <u>simple</u> formula for $y(t)$ when $f_s = 5000$ samples/sec. (for both C/D and D/C converters)



Problem 2

When we multiply two sinusoids having different frequencies, we can create an interesting audio effect called a *beat note*. We multiply the following two sinusoids:

$$x(t) = \cos\left(2\pi\left(40\right)t - \pi/3\right)\cos\left(2\pi\left(600\right)t + \pi/4\right)$$

a) Rewrite $x(t)$ as an additive set of complex exponential signals that sum together to make $x(t)$.
b) Plot the spectrum of $x(t)$.
c) Use the spectrum to write an alternative formula for $x(t)$ as:

$$x(t) = A\cos\left[2\pi\left(f_c - \Delta\right)t + \phi_1\right] + B\cos\left[2\pi\left(f_c + \Delta\right)t + \phi_2\right]$$

Find the numerical values for all the parameters: $A, B, f_c, \Delta, \phi_1$, and ϕ_2.
d) This signal is periodic; determine its fundamental period.

Problem 3

Consider the linear time-invariant (LTI) system whose impulse response is given by

$$h(n) = (-1)^n, n = 0, ..., L-1$$

a) Give an expression for the transfer function $H(z)$.
Hint: make use of the geometric series expansion

Let L = 4.
b) Determine the poles and zeros of $H(z)$ and plot them in the complex z-plane.
c) Determine the frequency response with magnitude and phase of the system.

d) Give a sketch of the magnitude and phase response for $-\pi \le \hat{\omega} < \pi$.

Consider the following three input signals:

$$x_1[n] = 2\cos\left(\frac{\pi}{3}\right)$$

$$x_2[n] = 5\cos\left(\frac{\pi}{2}n - \frac{\pi}{6}\right)$$

$$x_3[n] = \cos\left(\pi n + \frac{\pi}{4}\right)$$

e) Compute the corresponding output signals y_1, y_2 and y_3.

Consider the input signal

$$x[n] = 2x_1[n] + x_2[n-3] - 3x_3[n-1]$$

f) Compute the corresponding output $y[n]$.

Problem 4

Answer the following questions about the system whose z-transform system function is

$$H(z) = \frac{1+z^{-2}}{1+0.9z^{-1}}$$

(a) Determine and plot the poles and zeros of $H(z)$.

(b) Determine the difference equation relating the input and the output of this filter. What kind of filter is it?

(c) Derive a simple expression (purely real) for the magnitude squared of the frequency response $\left|H\left(e^{j\hat{\omega}}\right)\right|^2$.

(d) Evaluate the frequency response $\left|H\left(e^{j\hat{\omega}}\right)\right|^2$ at frequencies $\hat{\omega} = 0, \frac{\pi}{2}$ and π.

(e) Sketch the filter response. Is this a low pass or high pass filter? Explain your answer.

(f) If the input signal to the system is

$$x[n] = \delta[n] + 0.9\delta[n-1],$$

Determine the output signal $y[n]$ (Assume system at rest for n < 0).