Solutions written examination Signal Processing (

Problem 1 : a) Given: $x[n] = 10\cos(0.18\pi n + \pi/4)$

$\longrightarrow \quad x(t) = 10\cos(2\pi f_0 t + \varphi) \,/\, t \leftarrow \frac{n}{f_s}$

$$2\pi\frac{f_0}{f_s} = 0{,}18\pi \Longrightarrow f_0 = 0{,}09 f_s = 0.09 \cdot 2500 = \underline{225\,Hz}$$

This is the ordinary sampling case, so

$$x(t) = 10\cos\left(2\pi(225)t + \frac{\pi}{4}\right).$$

The other possibility would be the "Folding" case

$\rightarrow \quad x(t) = 10\cos(2\pi(-f_0 + f_s)t - \varphi) \,/\, t \leftarrow \frac{n}{f_s}$

$$\frac{2\pi(-f_0+f_s)}{f_s} = 0{,}18\pi \quad \rightarrow f_0 = \underline{2275}\ Hz,$$

Then $\quad x(t) = 10\cos(2\pi(2275)t - \pi/4)$

need phase reversal for folding.

b) Construct lowest frequency sinusoid

$\therefore \quad y(t) = 10\cos\left(2\pi(225)t + \frac{\pi}{4}\right)$; 225 Hz will be output frequency.

c) $x(t) = 2\cos(2\pi(11{,}000)t - 3\pi/4) + 4\cos\left(2\pi(3000)t - \frac{\pi}{4}\right)$

$$x[n] = 2\cos\left(2\pi\left(\frac{11.000}{5000}\right)n - \frac{3\pi}{4}\right) + 4\cos\left(2\pi\left(\frac{3000}{5000}\right)n - \frac{\pi}{4}\right)$$

$$x[n] = 2\cos\left(2\pi(2.2)n - \frac{3\pi}{4}\right) + 4\cos\left(2\pi(0.6)n - \frac{\pi}{4}\right)$$

$$x[n] = \underbrace{2\cos\left(2\pi(0.2)n - \frac{3\pi}{4}\right)}_{\text{sub sampling}} + \underbrace{4\cos\left(2\pi(-0.4)n - \frac{\pi}{4}\right)}_{\text{folding}}$$

$$4\cos(2\pi(0.4)n + \pi/4)$$

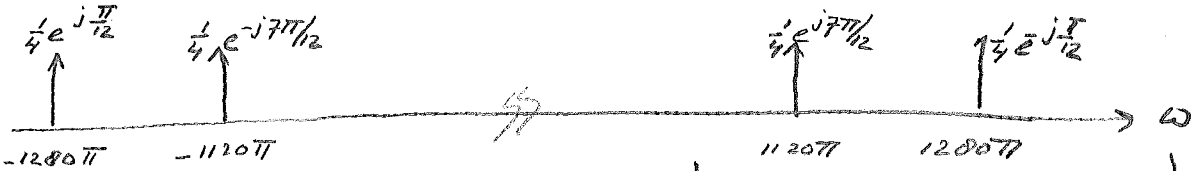
$\rightarrow \quad y(t) = 2\cos(2\pi(1000)t - 3\pi/4) + 4\cos(2\pi(2000)t + \pi/4)$. sign!

Problem 2 a) $x(t) = \cos(2\pi(40)t - \pi/3)\cos(2\pi(600)t + \pi/4)$

$$x(t) = \frac{1}{2}\cdot\frac{1}{2}\cdot\left[\left(e^{j80\pi t}\cdot e^{-j\pi/3} + e^{-j80\pi t}\cdot e^{j\pi/3}\right)\left(e^{j1200\pi t}\cdot e^{j\pi/4} + e^{-j1200\pi t}\cdot e^{-j\pi/4}\right)\right.$$

$$= \frac{1}{4}\left[e^{j1280\pi t}\cdot e^{-j\pi/12} + e^{j1120\pi t}\cdot e^{j7\pi/12} + e^{-j1120\pi t}\cdot e^{-j7\pi/12} + e^{-j1280\pi t}\cdot e^{j\ldots}\right.$$

b)

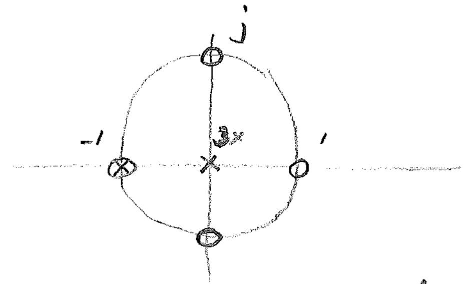


c) Using b) $\quad x(t) = \frac{1}{2}\cos\left(1280\pi t - \frac{\pi}{12}\right) + \frac{1}{2}\cos\left(1120\pi t + \frac{7\pi}{12}\right)$

d) Fundamental frequency, $\omega_0 = 160\pi \Longrightarrow$ Fund. Period, $T_0 = \frac{1}{80}$ se

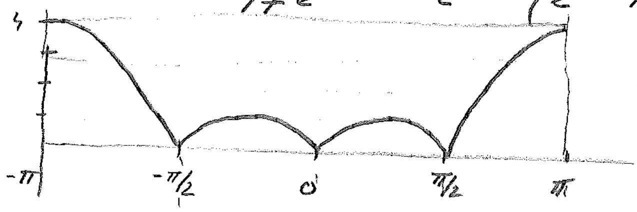
Problem 3 a) $H(z) = \sum_{n=0}^{L-1} (-1)^n z^{-n} = \sum_{n=0} \left(-z^{-1}\right)^n = \frac{1-\left(-z^{-1}\right)^L}{1+z^{-1}} = \frac{1-(-1)^L z^{-L}}{1+z^{-1}}$

b) $L=4$ $\quad H(z) = \frac{1-z^{-4}}{1+z^{-1}} = \frac{z^4-1}{z^3(z+1)}$; Hence, four zeros at $z = e^{j\frac{2\pi}{4}k} = e^{j\frac{\pi}{2}k}$, $k=0,\ldots,3$, three poles at $z=0$, and one pole at $z=-1$

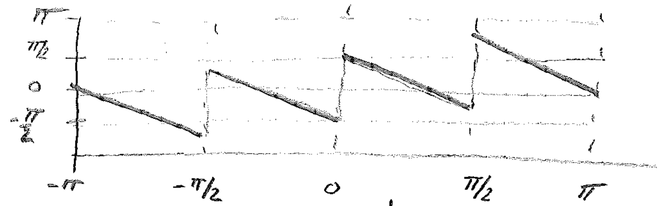


c) $H\left(e^{j\hat{\omega}}\right) = \frac{1-e^{-j4\hat{\omega}}}{1+e^{-j\hat{\omega}}} = \frac{e^{-j2\hat{\omega}}\left(e^{j2\hat{\omega}}-e^{-j2\hat{\omega}}\right)}{e^{-j\hat{\omega}/2}\left(e^{j\hat{\omega}/2}+e^{-j\hat{\omega}/2}\right)} = \frac{\sin(2\hat{\omega})}{\cos\left(\frac{\hat{\omega}}{2}\right)} \cdot e^{-j\left(\frac{3}{2}\hat{\omega}-\frac{\pi}{2}\right)}$

d)



magnitude resp



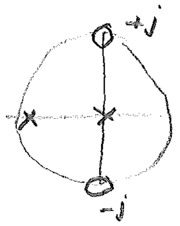
phase response

e) Since $H(e^{j0}) = H(e^{j\frac{\pi}{2}}) = 0$ and $H(e^{j\pi}) = 4$, we have
$y_1[n] = y_2[n] = 0$, and $y_3[3] = 4\cos\left(\pi n + \frac{\pi}{4}\right)$

f) Since the system is LTI, we have
$y[n] = -3 * 4\cos\left(\pi(n-1)+\frac{\pi}{4}\right) = -12\cos\left(\pi(n-1)+\frac{\pi}{4}\right)$

Problem 4

a) zeros when $1+z^{-2}=0$; $z=\pm j \Rightarrow e^{j\frac{\pi}{2}}$ and $e^{-j\frac{\pi}{2}}$
poles when $z=0, -0.9$



b) $y[n] = -0.9\,y[n-1] + x[n] + x[n-2]$, IIR filter

c) $\left|H\left(e^{j\hat{\omega}}\right)\right|^2 = H\left(e^{j\hat{\omega}}\right)H^*\left(e^{j\hat{\omega}}\right) = \frac{1+e^{-j2\hat{\omega}}}{1+0.9e^{-j\hat{\omega}}} \cdot \frac{1+e^{+j2\hat{\omega}}}{1+0.9e^{+j\hat{\omega}}} = \frac{2+2\cos\hat{\omega}}{1.81+1.8\cos\hat{\omega}}$

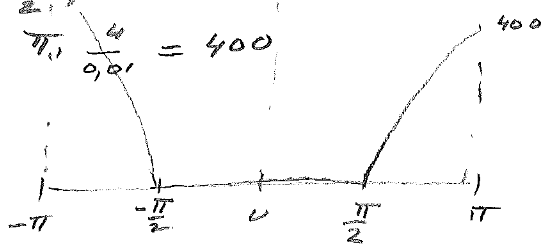
d) $\hat{\omega}=0$, $\frac{4}{3.61} = 1.11$

$\hat{\omega} = \frac{\pi}{2}$, $\quad 0$

$\hat{\omega} = \pi$, $\frac{4}{0.01} = 400$

High-pass filter,
high gain at $\hat{\omega} = \pm\pi$

e)



f) $Y(z) = H(z)X(z) = \frac{1+z^{-2}}{1+0.9z^{-1}} \cdot 1+0.9z^{-1} = 1+z^{-2}$

$y[n] = \delta[n] + \delta[n-2]$