DELFT UNIVERSITY OF TECHNOLOGY
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Information and Communication Theory Group

SIGNAL PROCESSING (IN2405 - I)

Written examination, Friday 30 January 2009 (14:00 - 17:00)

The problems are weighted equally in calculating the final grade. Therefore, try to spend your time wisely. Please, restrict yourself to the essence when answering “discussion type” of questions. You can answer in Dutch or English!

Please, start every problem on a **new** sheet of paper and write down your **name**. Good luck!

This exam contains 4 problems

Problem 1

a) Prove De Moivre’s formula:

$$(\cos\theta + j\sin\theta)^n = \cos n\theta + j\sin n\theta$$

b) Then use it to evaluate:

$$\left(\cos(\frac{\pi}{3}) + j\sin(\frac{\pi}{3})\right)^3$$

Give the answer in rectangular form.

c) Simplify the following expression:

$$z(t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{e^{j\omega_0 t} + e^{-j\omega_0 t}}$$

by giving a simple expression for the magnitude and phase of *z(t)*.

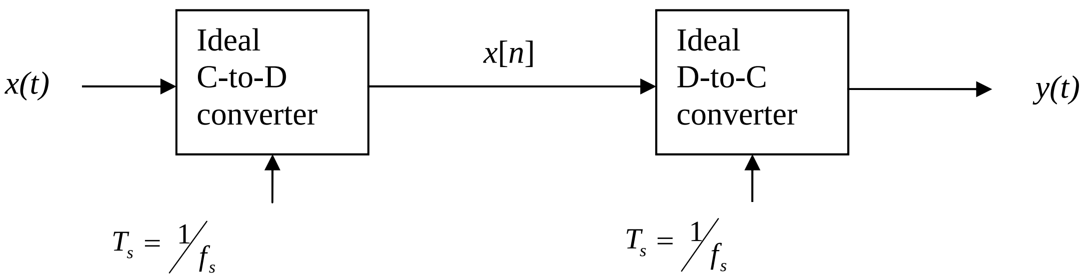
d) Solve for $w[n]$ in the following equation:

$$w[n] - \cos\left(8n + \frac{11\pi}{3}\right) = 2\cos\left(8n + \frac{7\pi}{3}\right)$$

Express $w[n]$ in the form $w[n] = A\cos(\hat{\omega}_0 n + \phi)$

e) Draw a vector diagram of the phasor addition to solve part *d*.

Problem 2



Consider the above system:

Let $x(t) = 13\sin(22\pi t)$. The discrete-time signal $x[n]$ is obtained by sampling *x(t)* at a rate f_s and can be written as

$$x[n] = A\cos(\hat{\omega}_0 n + \phi)$$

a) Let the sampling frequency be f_s = 10 samples/sec; determine the values of $A, \phi, \hat{\omega}_0$, determine and plot the discrete-time spectrum for $x[n]$ by labeling frequency, amplitude and phase of each spectral component and state whether or not the signal has been oversampled or under sampled (with or without folding)
b) Idem for f_s = 25 samples/sec.
c) Idem for f_s = 15 samples/sec.

The output of the ideal D-to-C converter is given by *y(t)*.
d) Determine the reconstruction frequency f_{rec} of *y(t)* for resp. f_s = 10, 25 and 15 samples/sec.

<u>Problem 3</u>
A discrete-time system is defined by the input/output relation

$$y[n] = -3x[n-2] + 6x[n-4] - 3x[n-6] \qquad (1)$$

a) Determine whether or not the system defined by Eq. (1) is (i) linear, (ii) time-invariant, (iii) causal. Explain your answers.
b) Draw a block diagram that represents this system in terms of unit delay elements, coefficients multipliers and adders.
c) Obtain an expression for the impulse response $h[n]$ of this system.
d) Obtain an expression for the frequency response $H(\hat{\omega})$ of this system.
e) Make a sketch of the frequency response (magnitude and phase) as a function of frequency.
Hint: Use symmetry to simplify your expression before determining the magnitude and phase.
f) For the system of Eq. (1), determine the output $y_1[n]$ when the input is:

$$x_1[n] = 4 + 8\cos(0.5\pi n + \pi/2)$$

Hint: Use the frequency response and superposition to solve this problem

<u>Problem 4</u>

Answer the following questions about the system whose z-transform system function is

$$H(z) = \frac{1 + z^{-2}}{1 + 0.77z^{-1}}$$

(a) Determine and plot the poles and zeros of $H(z)$.

(b) Determine the difference equation relating the input and the output of this filter. What kind of filter is it?

(c) Derive a simple expression (purely real) for the magnitude squared of the frequency response $\left|H\left(e^{j\hat{\omega}}\right)\right|^2$.

(d) Evaluate the frequency response $\left|H\left(e^{j\hat{\omega}}\right)\right|^2$ at frequencies $\hat{\omega} = 0, \frac{\pi}{2}$ and π.

(e) Sketch the filter response. Is this a low pass or high pass filter? Explain your answer.