Answers: Written exam Signal Processing (IN2405-I)
Friday 30 January 2009

1a. Using Euler's formula: $(\cos\theta + j\sin\theta)^n = (e^{j\theta})^n = e^{jn\theta}$
$= \cos n\theta + j\sin n\theta$

b. $\left(\cos\left(\frac{\pi}{3}\right) + j\sin\left(\frac{\pi}{3}\right)\right)^3 = \cos\pi + j\sin\pi = -1 + j0.$

c. Use Inverse Euler:

$$\sin(\omega_0 t) = \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j}; \quad \cos(\omega_0 t) = \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$z(t) = \frac{2j\sin(\omega_0 t)}{2\cos(\omega_0 t)} = j\frac{\sin(\omega_0 t)}{\cos(\omega_0 t)} = e^{j\frac{\pi}{2}} \cdot \tan(\omega_0 t)$$

phase is $\frac{\pi}{2}$; Magnitude is $\tan(\omega_0 t)$. Strictly speaking the magnitude is the absolute value of $\tan(\omega_0 t)$ since tangent can be negative!

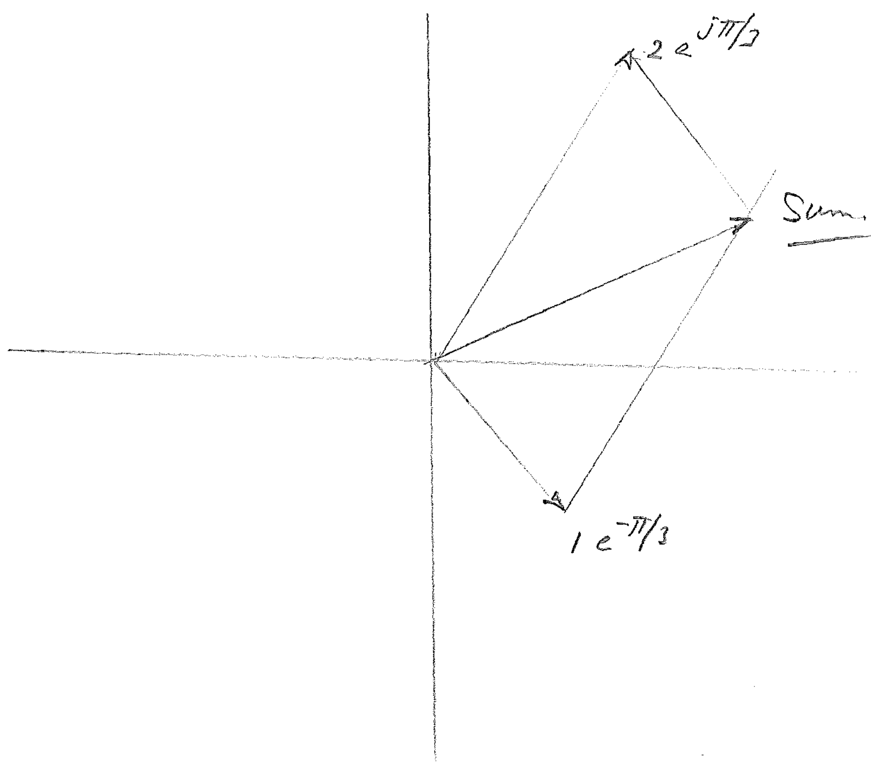
d. Convert to phasors: $1.e^{j11\pi/3} + 2.e^{j7\pi/3}$;

remove multiples of 2π: $1.e^{-j\pi/3} + 2.e^{j\pi/3}$

convert to rectangular form: $\frac{1}{2} - j.\frac{1}{2}\sqrt{3} + 2\left(\frac{1}{2} + j.\frac{1}{2}\sqrt{3}\right) = \frac{3}{2} + j.\frac{1}{2}\sqrt{3}$
$= \sqrt{3}e^{j\pi/6}$

$\Rightarrow w[n] = \sqrt{3}\cos\left(8n + \frac{\pi}{6}\right)$; $\hat{\omega}_0 = 8$, $A = \sqrt{3}$, $\varphi = \pi/6$

e.



2. $x(t) = 13 \sin 22\pi t$ → 11 Hz $\qquad x[n] = A\cos(\hat{\omega}_0 n + \phi)$ (2)

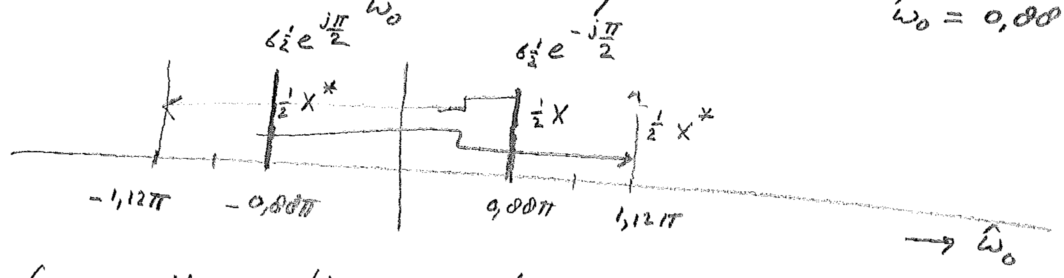
b) $f_s = 25$ samples/sec (Hz)

$f_s > 2.(11) \Rightarrow$ <u>Over sampling</u>

$x[n] = 13\cos\left(22\pi\left(\frac{n}{25}\right) - \frac{\pi}{2}\right)$

$= 13\cos\left(\underbrace{0{,}88\pi}_{\hat{\omega}_0}.n - \frac{\pi}{2}\right)$

$A = 13$
$\phi = -\frac{\pi}{2}$
$\hat{\omega}_0 = 0{,}88\pi$



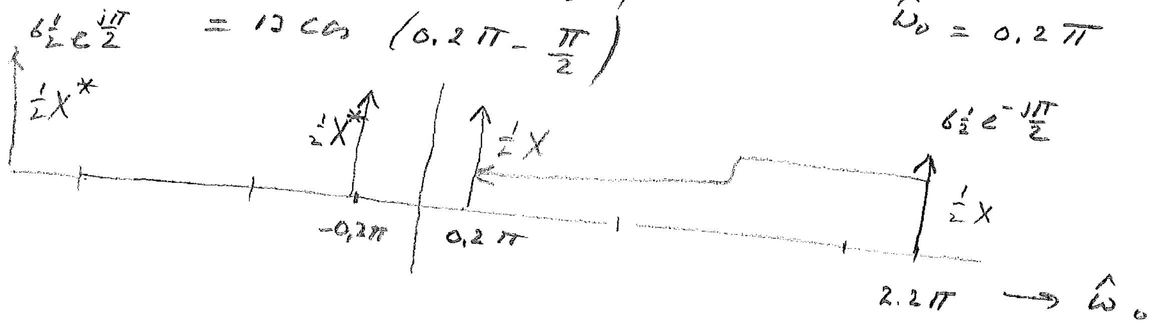
a) $f_s = 10$ Hz <u>Undersampling</u>

$x[n] = 13\cos\left(22\pi\left(\frac{n}{10}\right) - \frac{\pi}{2}\right)$

$= 13\cos\left(2.2\pi - \frac{\pi}{2}\right)$

$= 13\cos\left(0.2\pi - \frac{\pi}{2}\right)$

$A = 13$
$\phi = -\frac{\pi}{2}$
$\hat{\omega}_0 = 0.2\pi$



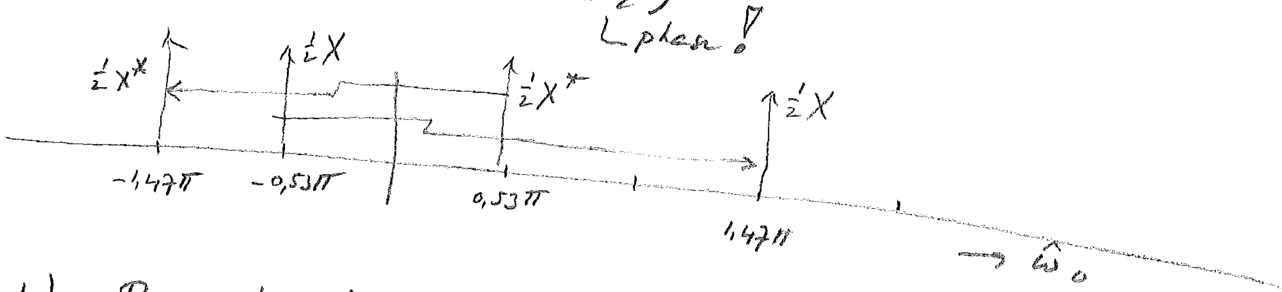
c) $f_s = 15$ Hz <u>under sampling</u> (folding)

$x[n] = 13\cos\left(22\pi\left(\frac{n}{15}\right) - \frac{\pi}{2}\right)$

$= 13\cos\left(1.47\pi.n - \frac{\pi}{2}\right)$

$= 13\cos\left(0.53\pi.n \; (+) \; \frac{\pi}{2}\right)$

↳ phase !



d) <u>Reconstruction</u>

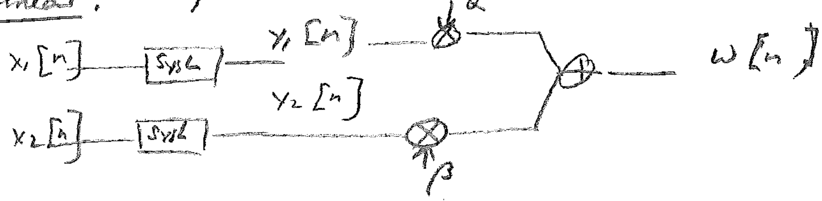
$f_s = 25$ Hz ; $\hat{\omega}_0 = \frac{2\pi\, freq}{f_s} = 0{,}88\pi$; $freq = \frac{0{,}88\pi}{2\pi}.25 = 11$ Hz

$f_s = 10$ Hz ; $\hat{\omega}_0 = \frac{2\pi\, freq}{f_s} = 0{,}2\pi$; $freq = \frac{0{,}2\pi}{2\pi}.10 = 1$ Hz

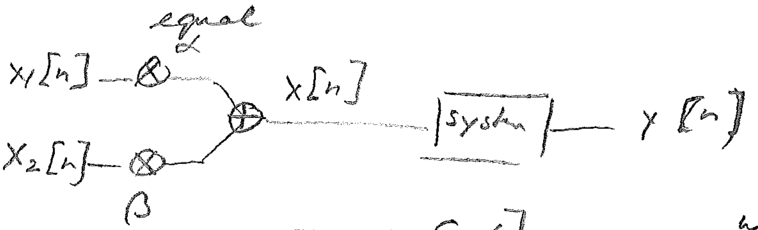
$f_s = 15$ Hz ; $\hat{\omega}_0 = \frac{2\pi\, freq}{f_s} = 0{,}53\pi$; $freq = \frac{0{,}53\pi}{}.15 = 4$ Hz

3.

a) System is linear: if $x[n] = \alpha x_1[n] + \beta x_2[n] \Rightarrow y[n] = \alpha y_1[n] + \beta y_2[n]$



if $w[n] = y[n]$



$w(n) = \alpha y_1[n] + \beta y_2[n]$

$y_1[n] = -3x_1[n-2] + 6x_1[n-4] - 3x_1[n-6]$

$y_2[n] = -3x_2[n-2] + 6x_2[n-4] - 3x_2[n-6]$

$x[n] = \alpha x_1[n] + \beta x_2[n]; \quad y[n] = \alpha y_1[n] + \beta y_2[n]$

time-invariant: $x[n-n_0] \rightarrow y[n-n_0]$

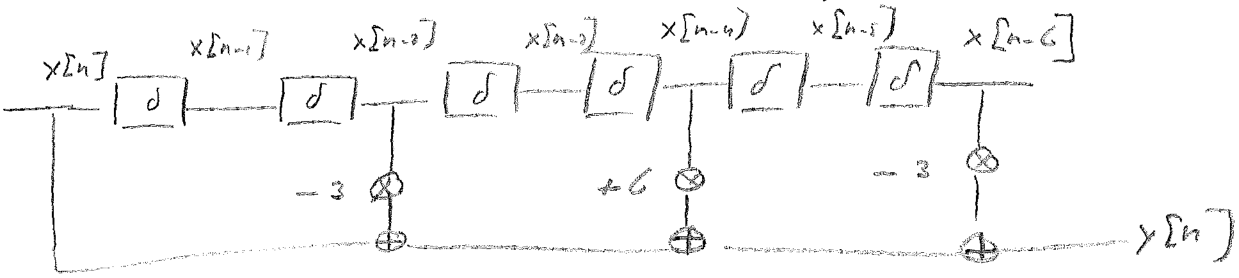
plug in $x[n-n_0]$ yields

$-3x[n-n_0-2] + 6x[n-n_0-4] - 3x[n-n_0-6] \equiv y[n-n_0]$

causal: $y[n]$ only depends on inputs previous to n: $x[n-2], x[n-4], x[n-6]$

$\Rightarrow$ causal system.

b).



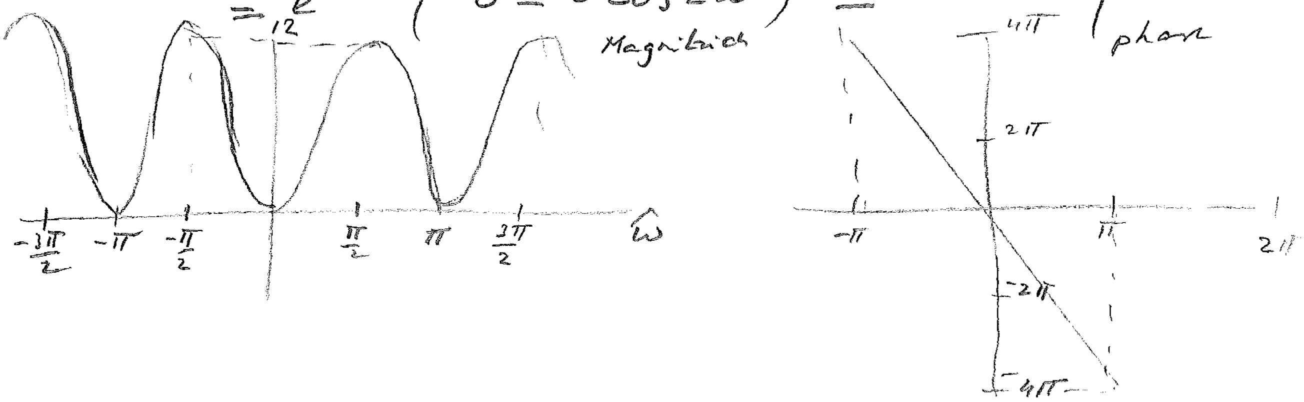
c) $h[n]$ is obtained by setting $x[n] = \delta[n]$:

$h[n] = -3\delta[n-2] + 6\delta[n-4] - 3\delta[n-6]$

d) $H(\hat{\omega}) = -3e^{-j2\hat{\omega}} + 6e^{-j4\hat{\omega}} - 3e^{-j6\hat{\omega}}$

e) $H(\hat{\omega}) = e^{-j4\hat{\omega}}\left(-3e^{j2\hat{\omega}} + 6 - 3e^{-j2\hat{\omega}}\right)$

$= e^{-j4\hat{\omega}}\left(6 - 6\cos 2\hat{\omega}\right) = 6e^{-j4\hat{\omega}}\left(1 - \cos 2\hat{\omega}\right)$



3 f) $x_1[n] = 4 + 8\cos(0.5\pi n + \pi/2)$

$x_1[n] = 4e^{j0 \cdot n} + 4e^{j\pi/2}e^{j0.5\pi n} + 4e^{-j\pi/2}e^{-j0.5\pi n}$

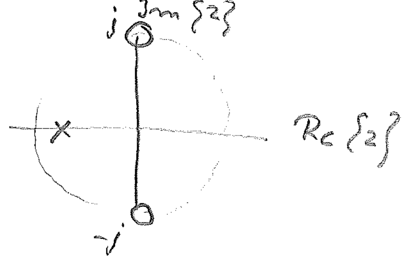
$y_1[n] = 4e^{j0n}H(0) + 4e^{j\pi/2}\cdot e^{j0,5\pi n}H(0,5\pi) + 4e^{-j\frac{\pi}{2}}\cdot e^{-j0.5\pi n}H(-0,5\pi)$

$= 4(0) + 4e^{j\pi/2}\cdot e^{j0,5\pi n}(12) + 4e^{-j\frac{\pi}{2}}\cdot e^{-j\cdot 0,5\pi n}(12)$

$= 96\cos(0,5\pi n + \frac{\pi}{2})$

4 a. zeros when $1+z^{-2}=0$ $\quad z=\pm j \Rightarrow e^{j\pi/2}$ and $e^{-j\pi/2}$

poles when $1+0.77z^{-1}=0 \to z=-0,77$



b. $y[n] = -0,77\,y[n-1] + x[n] + x[n-2]$

IIR-filter

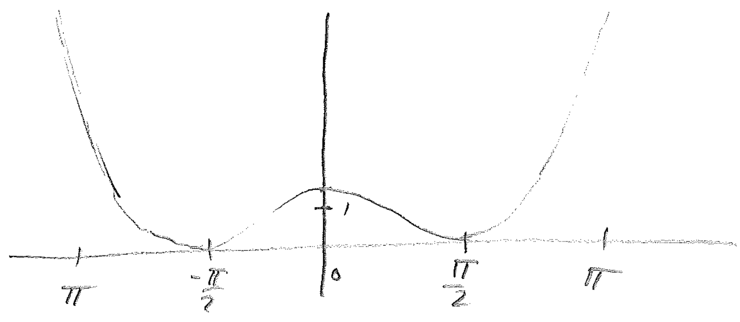
c. $|H(e^{j\hat\omega})|^2 = \frac{1+e^{-j2\hat\omega}}{1+0,77e^{-j\hat\omega}} \cdot \frac{1+e^{j2\hat\omega}}{1+0,77e^{j\hat\omega}} = \frac{2+2\cos(2\hat\omega)}{1,5929+1,54\cos(\hat\omega)}$

d. $\hat\omega = 0 \to \frac{2+2}{1,5929+1,54} \approx 1,277$

$\hat\omega = \pi/2 \quad = 0$

$\hat\omega = \pi \quad = 75,614$

e



Highpass, low frequencies are not that much magnified, and freq. $\pm\frac{\pi}{2}$ are nullified.