Answers Examination **Signal Processing**
(IN2405-1/ET2560IN)

October 31, 2006
(14:00 - 17:00)

Assignment 1:

Consider the following input signal to an ideal C-to-D converter:

$$x(t) = (4 + 2\cos(2\pi 100t))\, e^{j2\pi 50t}.$$

a) Using the inverse Euler formula, we have

$$\begin{aligned} x(t) &= (4 + 2\cos(2\pi 100t))\, e^{j2\pi 50t} \\ &= \left(4 + e^{j2\pi 100t} + e^{-j2\pi 100t}\right) e^{j2\pi 50t} \\ &= 4e^{j2\pi 50t} + e^{j2\pi 150t} + e^{-j2\pi 50t}. \end{aligned}$$

The spectrum trivially follows from this; three components at frequencies -50, 50 and 150 Hz.

b) The signal is periodic with period 1/50 = 0.02 s.

c) In order to avoid aliasing, we must have $f_s > 2f_{\max} = 300$ Hz.

d) Since $f_s > 2f_{\max}$ we have $y(t) = x(t)$.

e) The sampled signal $x[n]$ is given by

$$\begin{aligned} x[n] &= 4e^{j2\pi 50\frac{n}{250}} + e^{j2\pi 150\frac{n}{250}} + e^{-j2\pi 50\frac{n}{250}} \\ &= 4e^{j\frac{2\pi}{5}n} + e^{j\frac{6\pi}{5}n} + e^{-j\frac{2\pi}{5}n} \\ &= 4e^{j\frac{2\pi}{5}n} + e^{-j\frac{4\pi}{5}n} + e^{-j\frac{2\pi}{5}n}. \end{aligned}$$

Hence,

$$y(t) = 4e^{j2\pi 50t} + e^{-j2\pi 100t} + e^{-j2\pi 50t}.$$

f) The sampled signal $x[n]$ is given by

$$\begin{aligned} x[n] &= 4e^{j2\pi 50\frac{n}{50}} + e^{j2\pi 150\frac{n}{50}} + e^{-j2\pi 50\frac{n}{50}} \\ &= 4e^{j2\pi n} + e^{j6\pi n} + e^{-j2\pi n} \\ &= 4 + 1 + 1 = 6. \end{aligned}$$

Hence, $y(t) = 6$ for all t.

Assignment 2:

Consider the linear time-invariant system whose system function $H(z)$ is given by

$$H(z) = \left(1 - e^{j\frac{\pi}{2}} z^{-1}\right)\left(1 - e^{j\pi} z^{-1}\right)\left(1 - e^{j\frac{3\pi}{2}} z^{-1}\right).$$

a) The zeros are: $z = e^{j\frac{\pi}{2}}$, $z = e^{j\pi}$, $z = e^{j\frac{3\pi}{2}}$. The poles are: $z = 0$ (3 times). The system is an FIR system and is therefore stable.

b) The frequency response is

$$H(e^{j\hat{w}}) = \sum_{k=0}^{3} e^{-j\hat{\omega}k} = \frac{\sin(2\hat{\omega})}{\sin(\hat{\omega}/2)} e^{-\frac{3}{2}\hat{\omega}}.$$

c) Plots of $|H(e^{j\hat{w}})|$ and $\angle H(e^{j\hat{w}})$ for $-\pi \leq \omega \leq \pi$.

Consider the following inputs to the system:

$x_1[n] = 2 + \cos(\pi n - \frac{\pi}{27})$.

$x_2[n] = 3\cos(\frac{\pi}{2}n - \frac{\pi}{4})$.

$x_3[n] = 5\cos(\frac{2\pi}{3}n)$.

d) The corresponding outputs are

$$y_1[n] = 8.$$
$$y_2[n] = 0.$$
$$y_3[n] \approx 5\cos(\tfrac{2\pi}{3}n).$$

Consider the input $x[n] = 3\delta[n] - 3\delta[n-3]$.

e) The corresponding output is

$$y[n] = 3(\delta[n] + \delta[n-1] + \delta[n-2] - \delta[n-4] - \delta[n-5] - \delta[n-6]).$$

Consider a suddenly applied input $x[n]$ whose z-transform is given by

$$X(z) = \frac{1}{1 - e^{j\pi} z^{-1}}.$$

f) The output is

$$y[n] = \delta[n] + \delta[n-2].$$

Assignment 3:

A linear time-invariant (LTI) system has a frequency response given by:

$$H(e^{j\hat{\omega}}) = \frac{1 - 3e^{-j\hat{\omega}}}{1 - \frac{1}{2}e^{-j\hat{\omega}}}.$$

a) The corresponding system function is

$$H(z) = \frac{1 - 3z^{-1}}{1 - \frac{1}{2}z^{-1}}.$$

b) The zero is $z = 3$.
The pole is $z = 1/2$.
Since the pole is located inside the unit circle in the complex z-plane, the system is stable.

Assume the input to the system is $x[n] = \delta[n]$.

c) The corresponding output is the impulse response given by

$$h[n] = \left(\frac{1}{2}\right)^n u[n] - 3\left(\frac{1}{2}\right)^{n-1} u[n-1].$$

Assume the input to the system is $x[n] = 2\delta[n] - 3\delta[n-1]$.

d) Using superposition, the corresponding output is

$$y[n] = 2h[n] - 3h[n-1].$$

Assume that $x[n] = e^{j\frac{\pi}{2}n}u[n]$ is a suddenly applied input.

e) Using partial fraction expansion of $Y(z)$, we find

$$y[n] = \underbrace{A\left(\frac{1}{2}\right)^n u[n]}_{transient} + \underbrace{Be^{j\frac{\pi}{2}n}u[n]}_{steady-state},$$

with

$$A = -\frac{5}{1 - 2e^{j\frac{\pi}{2}}} \text{ and } B = \frac{1 - 3e^{-j\frac{\pi}{2}}}{1 - 0.5e^{j\frac{\pi}{2}}}.$$