Answers Examination **Signal Processing** (ET2560IN)

November 1, 2005

(9:00 - 12:00)

Assignment 1:

a)

$$\begin{aligned} x(t) &= 5\cos(2\pi 20t) + 2\cos(2\pi 70t)\cos(2\pi 20t) \\ &= 5\cos(2\pi 20t) + 2\left(\frac{e^{j2\pi 70t} + e^{-j2\pi 70t}}{2}\right)\left(\frac{e^{j2\pi 20t} + e^{-j2\pi 20t}}{2}\right) \\ &= 5\cos(2\pi 20t) + \cos(2\pi 50t) + \cos(2\pi 90t). \end{aligned}$$

b) Straightforward with answer from a)

c) Since $200 = f_s > 2f_{max} = 180$, we have $y(t) = x(t)$.

d) For $f_s = 80$ Hz, aliasing occurs for all terms with frequency above 40 Hz, which is the case for the two sinusoids at frequencies 50 and 90 Hz.

Let $x_1(t) = \cos(2\pi 50t)$ and $x_2(t) = \cos(2\pi 90t)$.

Sampling $x_1(t)$ gives $\hat{\omega} = \frac{2\pi 50}{80} > \pi$. Aliases are given by

$$\begin{aligned} \hat{\omega} &= \frac{5\pi}{4} + 2\pi l, \quad l \in \mathbb{Z}, \\ \hat{\omega} &= -\frac{5\pi}{4} + 2\pi l, \quad l \in \mathbb{Z}. \end{aligned}$$

Hence the principal alias is $\hat{\omega} = \frac{3\pi}{4}$ (folded). This gives $x_1[n] = \cos(\frac{3\pi}{4}n)$ and hence $y_1(t) = \cos(\frac{3\pi}{4}80t) = \cos(2\pi(30)t)$.

Sampling $x_2(t)$ gives $\hat{\omega} = \frac{2\pi 90}{80} > \pi$. Aliases are given by

$$\begin{aligned} \hat{\omega} &= \frac{9\pi}{4} + 2\pi l, \quad l \in \mathbb{Z}, \\ \hat{\omega} &= -\frac{9\pi}{4} + 2\pi l, \quad l \in \mathbb{Z}. \end{aligned}$$

Hence the principal alias is $\hat{\omega} = \frac{\pi}{4}$. This gives $x_2[n] = \cos(\frac{\pi}{4}n)$ and hence $y_2(t) = \cos(\frac{\pi}{4}80t) = \cos(2\pi 10t)$.

Alternatively, we have

$$\begin{aligned} x_1[n] &= \cos(2\pi 50 \frac{n}{80}) = \cos(\frac{5\pi}{4} n) \\ &= \cos(-\frac{5\pi}{4} n) = \cos(-\frac{5\pi}{4} n + 2\pi) = \cos(\frac{3\pi}{4} n). \end{aligned}$$

Hence, $y_1(t) = \cos(\frac{3\pi}{4} 80t) = \cos(2\pi 30t)$.
Similarly we have,

$$\begin{aligned} x_2[n] &= \cos(2\pi 90 \frac{n}{80}) = \cos(\frac{9\pi}{4} n) \\ &= \cos(\frac{9\pi}{4} n - 2\pi) = \cos(\frac{\pi}{4} n). \end{aligned}$$

Hence, $y_2(t) = \cos(\frac{\pi}{4} 80t) = \cos(2\pi 10t)$.

We conclude $y(t) = 5\cos(2\pi 20t) + \cos(2\pi 30t) + \cos(2\pi 10t)$. From this expression the spectrum sketch is straightforward.

Assignment 2:

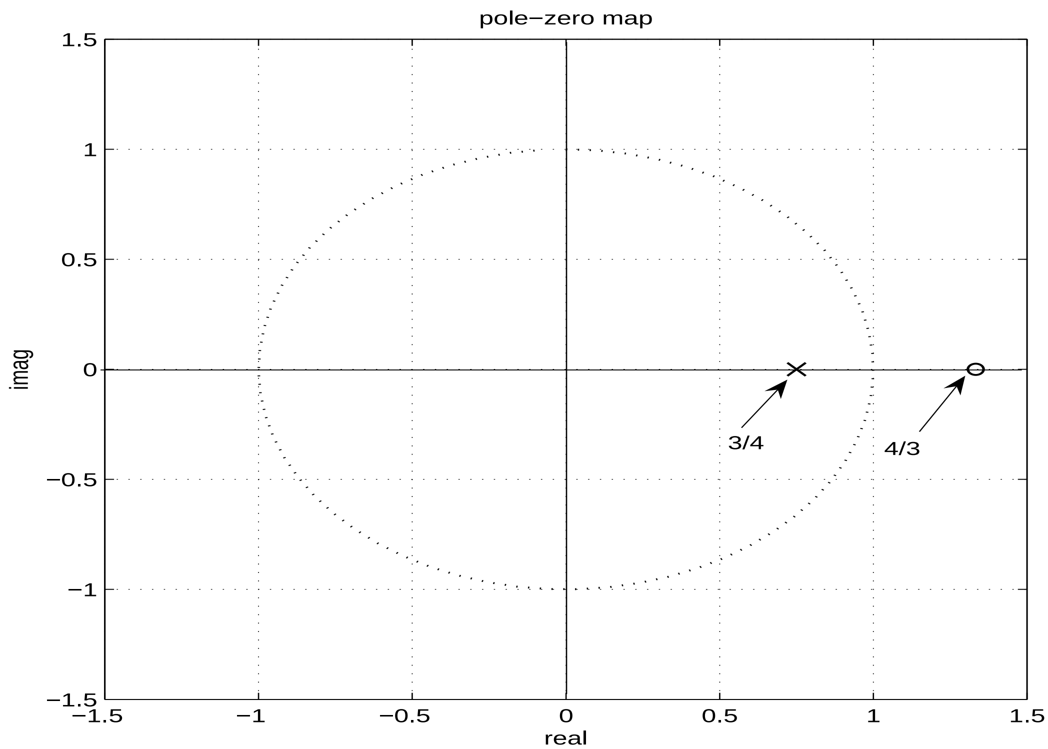
a)

$$H(z) = \sum_{n=0}^{L-1} (-1)^n z^{-n} = \sum_{n=0}^{L-1} (-z^{-1})^n = \frac{1-(-z^{-1})^L}{1+z^{-1}} = \frac{1-(-1)^L z^{-L}}{1+z^{-1}}$$

b)

$$H(z) = \frac{1-z^{-4}}{1+z^{-1}} = \frac{z^4-1}{z^3(z+1)}.$$

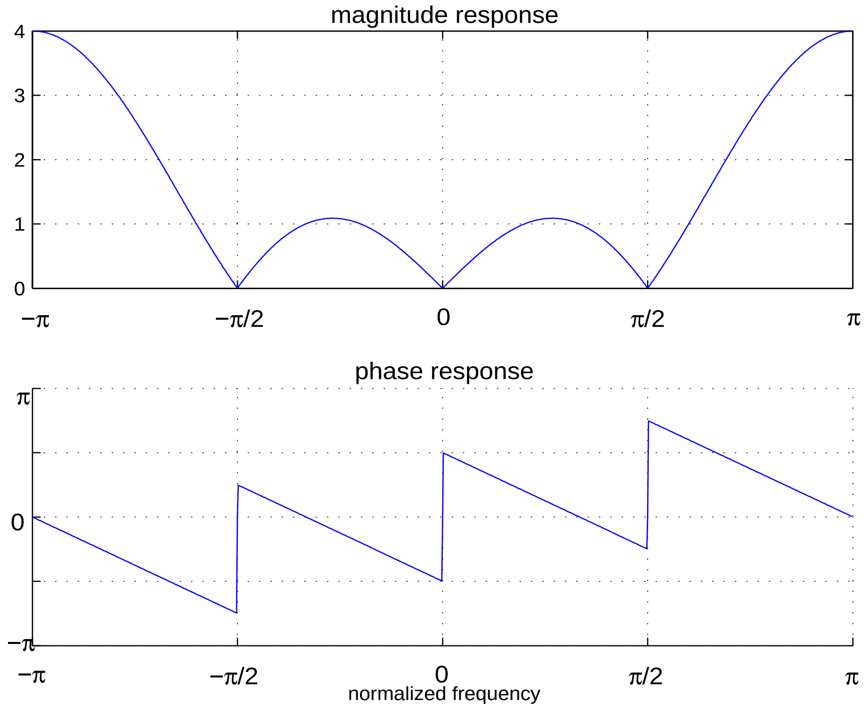
Hence, four zeros at $z = e^{j\frac{2\pi}{4}k} = e^{j\frac{\pi}{2}k}, k = 0, \ldots, 3$, three poles at $z = 0$ and one pole at $z = -1$.



c)

$$H(e^{j\hat{\omega}}) = \frac{1-e^{-j4\hat{\omega}}}{1+e^{-j\hat{\omega}}} = \frac{e^{-j2\hat{\omega}}(e^{j2\hat{\omega}} - e^{-j2\hat{\omega}})}{e^{-j\hat{\omega}/2}(e^{j\hat{\omega}/2} + e^{-j\hat{\omega}/2})} = \frac{\sin(2\hat{\omega})}{\cos(\hat{\omega}/2)} e^{-j(\frac{3}{2}\hat{\omega} - \frac{\pi}{2})}$$

d)



e) Since $H(e^{j0}) = H(e^{j\frac{\pi}{2}}) = 0$ and $H(e^{j\pi}) = 4$, we have

$$y_1[n] = y_2[n] = 0$$

and

$$y_3[n] = 4\cos(\pi n + \frac{\pi}{4}).$$

f) Since the system is LTI we have

$$y[n] = -3 * 4\cos(\pi(n-1) + \frac{\pi}{4}) = -12\cos(\pi(n-1) + \frac{\pi}{4}).$$

Assignment 3:

a) Using the coefficients from the difference equation we find

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 - 4z^{-1}}{1 + 0.9z^{-1}}.$$

b) Zeros: $z = 4/3$.
Poles: $z = -9/10$.
The system is stable because the pole is located inside the unit circle.

c) $X[n] = \delta[n]$:

$$y[n] = h[n] = \begin{cases} 0 & n < 0 \\ 3 & n = 0 \\ (3 + 4\frac{10}{9})(-0.9)^n & n > 0. \end{cases}$$

d) The input $x[n] = 2e^{j\frac{\pi}{3}n}u[n]$ is of the standard form $a^n u[n] \overset{\mathcal{Z}}{\leftrightarrow} \frac{1}{1-az^{-1}}$. Setting $a = e^{j\frac{\pi}{3}}$, we find

$$X(z) = \frac{2}{1 - e^{j\frac{\pi}{3}}z^{-1}}.$$

e)

$$Y(z) = H(z)X(z) = \frac{3 - 4z^{-1}}{1 + 0.9z^{-1}} \frac{2}{1 - e^{j\frac{\pi}{3}}z^{-1}}.$$

f) We use partial fraction expansion to find the coefficents A, and B in

$$Y(z) = \frac{A}{1 + 0.9z^{-1}} + \frac{B}{1 - e^{j\frac{\pi}{3}}z^{-1}}.$$

We find

$$A = Y(z)(1 + 0.9z^{-1})|_{z=-0.9} = \frac{14\frac{8}{9}}{1 + \frac{10}{9}e^{j\frac{\pi}{3}}},$$

and

$$B = Y(z)(1 - e^{j\frac{\pi}{3}z^{-1}})|_{z=e^{j\pi/3}} = \frac{6 - 8e^{-j\frac{\pi}{3}}}{1 + 0.9e^{-j\frac{\pi}{3}}}.$$

Using the inverse z-transform of each term we find

$$y[n] = \underbrace{A(-0.9)^n u[n]}_{\text{transient}} + \underbrace{Be^{j\frac{\pi}{3}n}u[n]}_{\text{steady state}}.$$